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ANALYSIS OF DEVIATION SHAPES ON NOISE-CAUSING GEARS

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Abstract

In modern industrial applications, gears serve as pivotal transmission components, whose transmission performance is critical to the operational stability of mechanical systems. For electric vehicles subject to stringent noise control specifications, the suppression and control of gear transmission noise carry heightened importance. To tackle gear noise at its root cause, this paper regards gear deviations as the primary excitation source of vibration and noise, and systematically analyzes six typical gear deviation shapes as well as the corresponding tooth-pair change-over characteristics in meshing. In addition, the formation mechanism of the corner contact phenomenon and its influential effect on gear transmission errors are deeply investigated. On this basis, a novel least-squares sine wave fitting method is proposed in this paper. The results demonstrate that the proposed method can effectively achieve the separation and quantitative characterization of diverse gear deviations. Meanwhile, this paper conducts an in-depth analysis of transmission errors in noise-causing gears, verifying that eccentricity error, pitch deviation and tooth surface texture of gears are the key influencing factors dominating transmission errors and subsequent noise excitation. This research provides a refined analytical paradigm and reliable technical support for the precision quality evaluation and noise fault diagnosis of gears.

Keywords: cylindrical gear, metrology, gear deviation, transmission errors.

1. Introduction

Gears are essential components for transmitting motion and power, and they play a critical role across a wide range of industrial applications as well as in daily life [1]. Their importance stems from their ability to achieve efficient and precise transmission, ensuring that complex machinery and equipment function smoothly and reliably. The performance of gears directly influences not only the operational smoothness of the system but also its long-term stability, service life, and energy efficiency [2]. In modern engineering applications, particularly where compactness and high efficiency are required, gears remain indispensable due to their high load-carrying capacity, reliability, and versatility.

In recent years, the rapid development of new energy vehicles has placed increasingly stringent requirements on the performance of transmission systems. In particular, the rising demands in the field of new energy vehicles have led to stricter requirements for gear vibration and noise control [3, 4]. To address these challenges, gear topology modification has emerged as an effective and widely adopted solution. Topology modification refers to the deliberate adjustment of gear tooth profiles and lead to optimize load distribution, minimize localized stress, and improve meshing behavior. This technology not only enhances the transmission performance of gears but also significantly reduces vibration, impact, and dynamic transmission error during operation [5]. Consequently, gear topology modification has become a critical tool in modern gearbox design, particularly for applications requiring high precision and low noise.

Among the various challenges in gearbox engineering, gear noise is one of the issues of greatest concern. Excessive gear noise not only affects user comfort but can also serve as an

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indicator of underlying mechanical defects that may compromise system reliability. Gear noise primarily originates from impacts between the teeth of driving and driven gears during meshing, as well as from resonance caused by periodic errors on tooth flanks [6-8]. In essence, gear deviations - including profile errors, pitch deviations, lead deviations, and surface waviness - are the main causes of such noise. When gear deviations are not effectively controlled, they can lead to unpredictable dynamic behavior, resulting in excessive vibration, unstable operation, and in severe cases, premature system failure [9-11]. Consequently, the inspection and analysis of gear deviations are critical. Diagnostic results not only provide timely feedback for optimizing the manufacturing process but also enable accurate predictions of gear performance, ensuring the system's reliable and stable operation [12].

Considerable research has been conducted worldwide regarding the effect of gear deviations on vibration and noise. For instance, Gravel proposed a method to identify noisy gears by focusing on ghost orders in the ripple signal [13]. This approach highlighted the connection between ripple patterns and gear noise characteristics, offering a pathway for diagnosing noise-related defects. Building on this, simulation-based ripple analysis has been developed to compare measured and calculated spectra, thereby enabling the identification of excitations introduced during the machining process and pinpointing the underlying causes of noise problems [14]. Furthermore, an extended methodology allows rapid calculation of vibration effects on the resulting gear surfaces, which provides a valuable means to assess the influence of process variations in advance and to optimize manufacturing strategies before mass production [15, 16]. These methods have been successfully implemented in industrial practice, such as within the Klingelnberg Gear Measuring Machine, which has become a benchmark tool for analyzing deviations and their impact on noise.

Nevertheless, existing approaches still have limitations. The majority of these techniques are particularly well suited for crowned gears, which represent only a subset of practical gear applications. In real engineering scenarios, various shapes of gear deviations are encountered, including complex combinations of profile, pitch, and surface waviness errors, which may exhibit different mechanisms of noise excitation. A comprehensive understanding of these deviation shapes and their relationship with noise generation remains a pressing need. Therefore, this paper presents an in-depth analysis of a broader range of deviation patterns in noise-causing gears. By systematically examining the spectral characteristics, transmission error contributions, and excitation mechanisms associated with different deviation shapes, the work aims to provide a more complete framework for diagnosing gear noise and guiding both manufacturing improvement and design optimization.

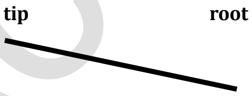
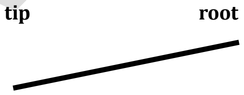
2. Gear deviation and tooth-pair change-over modes

Gear deviation refers to the deviation between the actual profile of a gear and its theoretically designed profile [17]. This deviation often arises from various factors during the gear manufacturing process, including machine tool inaccuracies, tool wear, fixture misalignment, thermal deformation, and even operator errors. This paper discusses gear profiles as examples, with similar analysis applied to the helix.

2.1. Gear deviation shapes

In the actual transmission, the gear deviation shape can be divided into six types, as illustrated in Table 1, including ideal, sinusoidal, crowning, concave, negative pressure angle, and positive pressure angle. The ideal profile maintains perfect theoretical geometry for optimal meshing. Sinusoidal deviation creates a wavy pattern along the tooth flank, causing periodic transmission errors. Crowning features a convex profile across the tooth width to evenly distribute contact stress. Concave deviation forms a hollow curvature, leading to stress concentration. Negative pressure angle deviation inclines the tooth tip outward, reducing the effective pressure angle. Positive pressure angle deviation slopes the tooth tip inward, increasing the pressure angle. These distinct deviation patterns significantly impact transmission accuracy and noise characteristics.

Table 1. Results of numerical experiments for stationary and nonstationary filters.

Number	**Deviation**	**Deviation chart**
1	Ideal	
2	Sinusoidal	
3	Crowning	
4	Concave	
5	Negative Pressure angle	
6	Positive Pressure angle	

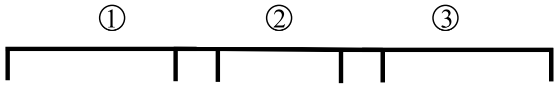
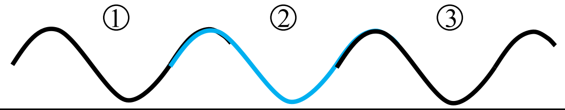
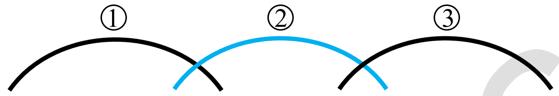
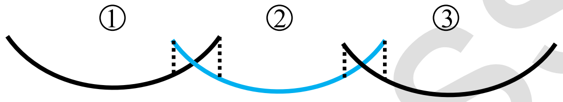
2.1. Gear tooth-pair change-over modes

During gear transmission, when one pair of teeth change to following another, a common meshing zone theoretically exists [18]. In this zone, the simultaneous meshing of the previous and next pairs of teeth allows for a smooth transfer of load from the previous pair to the next. When gears exhibit deviation, the smooth tooth-pair change-over between gear pairs is disturbed. Based on the six typical types of gear deviation shapes, gear tooth-pair change-over modes can be proposed, as shown in Table 2.

3. Transmission error and gear deviation analysis

Transmission error refers to the difference between the actual position of the output and the ideal position that the output shaft of a drive would occupy if the drive were perfect [19-21]. Gear noise is not only influenced by the amplitude of transmission error but is also closely related to the tooth-pair change-over modes of the gear. According to Gear Integrated Error Theory [22], the outer envelope curve of the gear tooth-pair change-over process represents transmission error. As shown in Table 2, the transmission errors in the case of multiple change-over, acceleration change-over and deceleration change-over exhibit “jump” phenomenon theoretically. Actually, due to base pitch deviation, the gear undergoes a corner contact process instead of directly entering the involute meshing [23].

Table 2. Six typical types of gear tooth-pair change-over modes.

Number	Type	Gear tooth pair change over modes
1	Ideal	
2	Sinusoidal	
3	Crowning	
4	Concave	
5	Negative Pressure angle	
6	Positive Pressure angle	

The transmission error during the acceleration change-over [24] is expressed as:

$$f\ \ (\varphi) = \begin{cases} (2Q\theta - u)\varphi - Q\varphi^2 & (0 \le \varphi \le \theta) \\ u(\gamma - \varphi) & (\theta < \varphi \le \gamma)' \end{cases} \tag{1}$$

where: θ - the corner contact angle, u - the constant value related to the gear parameters, Q - the constant value related to the gear parameters, γ - the standard pitch angle.

The transmission error during the deceleration change-over is expressed as:

$$\varphi) = \begin{cases} u^*\varphi & (0 \le \varphi \le \lambda - \theta) \\ u^*\varphi - Q^*(\varphi - \gamma + \theta)^2 & (\lambda - \theta < \varphi \le \gamma) \end{cases} \tag{2}$$

where: u^* - the constant value related to the gear parameters, Q^* - the constant value related to the gear parameters.

The transmission errors during acceleration change-over and deceleration change-over are shown in Fig. 1.

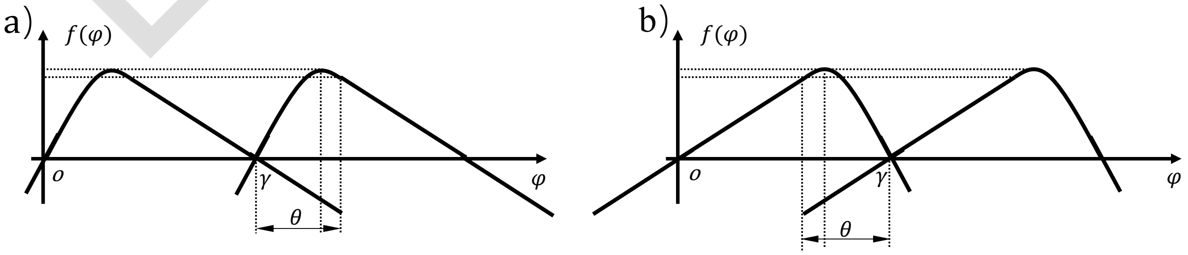
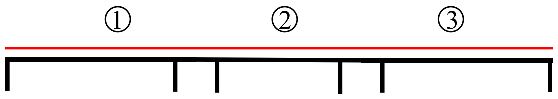
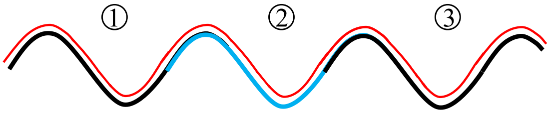
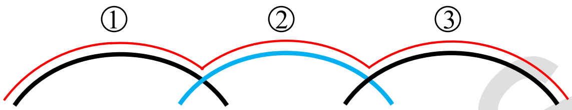
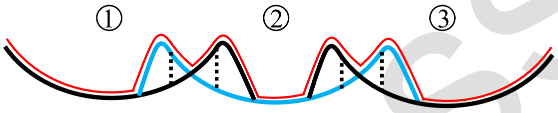
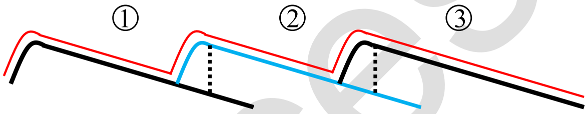
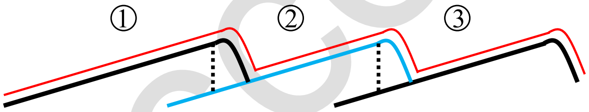


Fig. 1. Transmission errors with change-over: a) acceleration change-over, b) deceleration change-over.

During the corner contact, the shape of transmission error is a parabola instead of “jump”. Based on the above analysis, the transmission errors for multiple change-over, acceleration change-over and deceleration change-over should be modified, as illustrated in Table 3.

Table 3. Six typical types of gear tooth-pair change-over modes.

Number	Type	Transmission errors (red)
1	Ideal change-over	
2	Smooth change-over	
3	Tooth flank change-over	
4	Multiple change-over	
5	Acceleration change-over	
6	Deceleration change-over	

For separating gear deviations, this paper adopts a method based on least squares fitting of a sine wave function. By identifying the wavelength, amplitude and phase of the sine function, the periodic components in the gear deviations are extracted.

The mathematical model for fitting is defined as follows:

$$\sum_{j=1}^{N}\left(y_j - A_k \cdot sin\left(2\pi \cdot \frac{x_j}{\lambda_k} + \varphi_k\right)\right)^2 \to \underset{A_k,\lambda_k,\varphi_k}{\mathrm{Min}}\,, \tag{3}$$

where: λ_k - the wavelength, A_k - the amplitude corresponding to a wavelength of λ_k, φ_k - the initial phase corresponding to a wavelength of λ_k, x_j - the rotation angle, y_j - the transmission error value.

The spectral analysis results are shown in Fig 2, where the "meshing orders" are in blue and the "ghost orders" are in red.

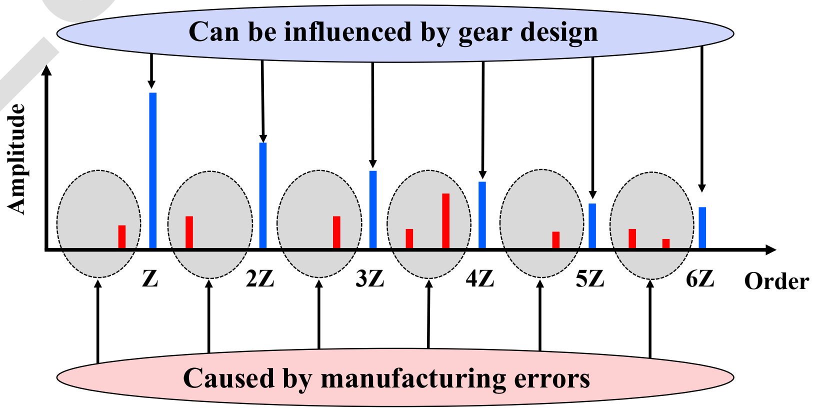


Fig. 2. Spectral analysis results.

4. Analysis of gear transmission error

Transmission error is a key factor affecting gear noise and performance, and it is closely related to various forms of gear deviations. This section analyzes the transmission error characteristics under different deviation conditions, including tooth-pair change-over, eccentricity and pitch deviation, as well as irregular tooth surface texture.

4.1. Analysis of typical gear transmission errors

The transmission errors shown in Fig. 3 are analyzed. The parameters of gear are teeth number 21, module 1.658 mm , and pressure angle 20°. The amplitude of profile deviation for each gear is identical. For the ideal change-over, the transmission error contains no harmonic components, as shown in Figure 3a. The smooth change-over exhibits only the component corresponding to the 1st meshing order, as shown in Figure 3b.

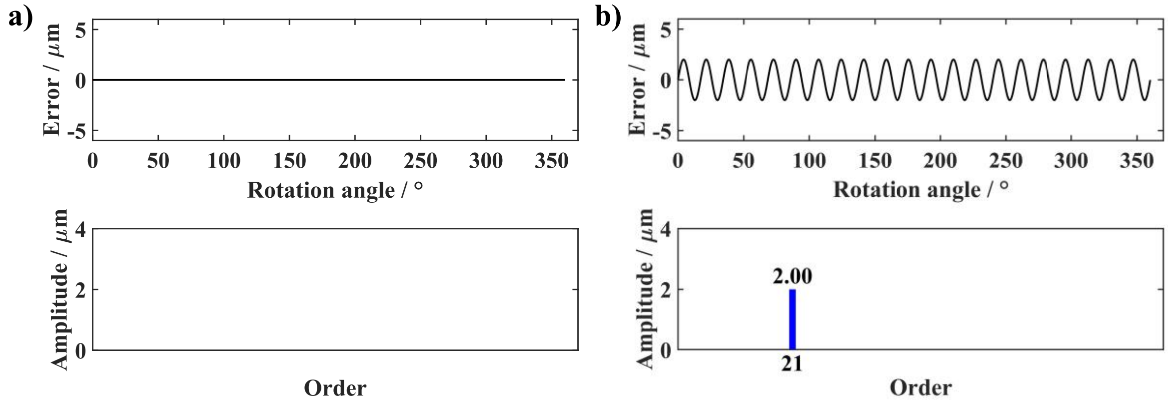


Fig. 3. Analysis of typical transmission errors: a) ideal change-over, b) smooth change-over.

The tooth flank change-over includes only the multiples of meshing orders, as shown in Fig. 4a. In the multiple change-over, its spectral analysis contains meshing orders with larger amplitudes, as shown in Fig. 4b.

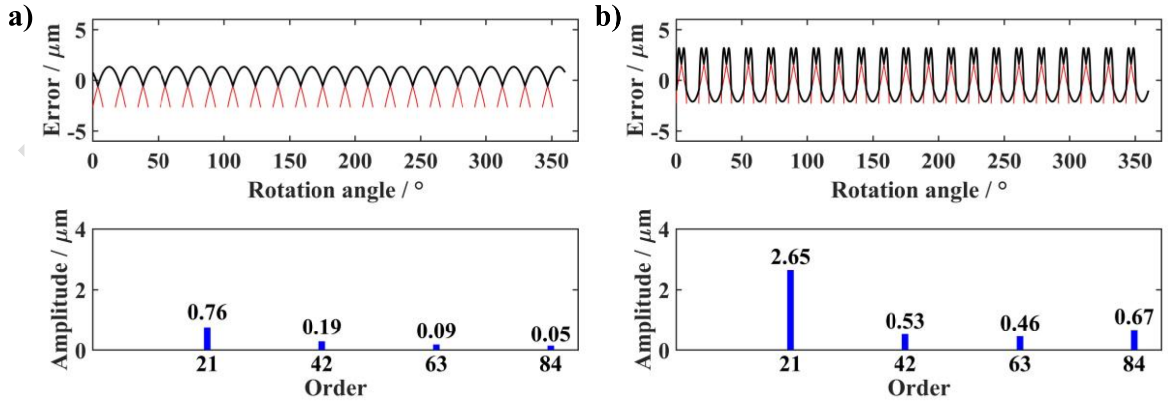


Fig. 4. Analysis of typical transmission errors: a) tooth flank change-over, b) multiple change-over.

For both acceleration change-over and deceleration change-over, the spectral components show identical orders and amplitudes, as shown in Fig. 5a and Fig. 5b, despite the different transmission errors.

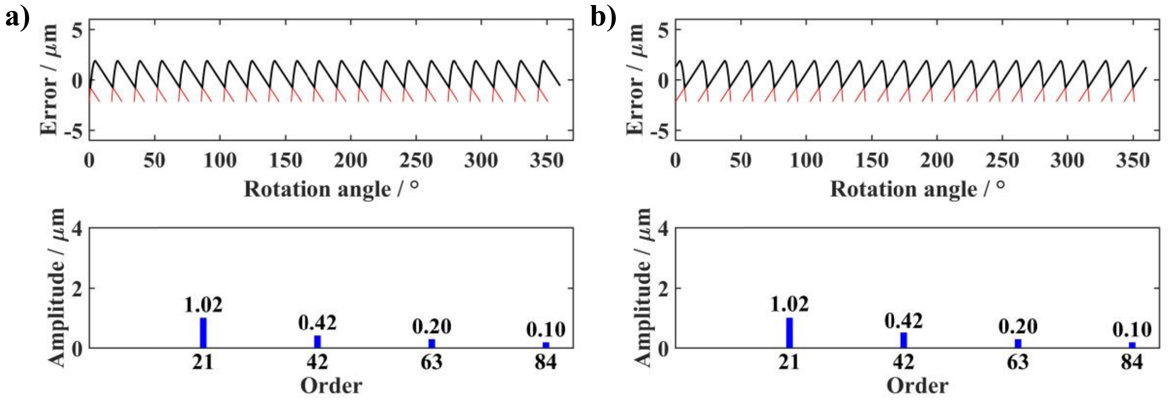


Fig. 5. Analysis of typical transmission errors with change-over: a) acceleration change-over, b) deceleration change-over.

4.1. Analysis of typical gear transmission errors

Figure 6a presents the analysis results under an eccentricity-free condition, which yields a smooth transmission error curve with frequency content exclusively at the meshing order and its harmonics. Figure 6b shows a transmission error with eccentricity, where the component of the 1st order represents the radial runout caused by gear eccentricity. If the radial runout contains multiple sine waveforms, it will extend along the tooth flanks meshing order to both sides, forming sidebands at both ends of the meshing order, as shown in Fig. 6c. Figure 6d demonstrates the effect of pitch deviation, resulting from grinding wheel wear, which leads to a gradual thickening of the gear profile. During grinding, the transition from the last tooth slot to the first introduces significant pitch deviation, generating multiple harmonic components in the spectrum instead of the traditional sidebands.

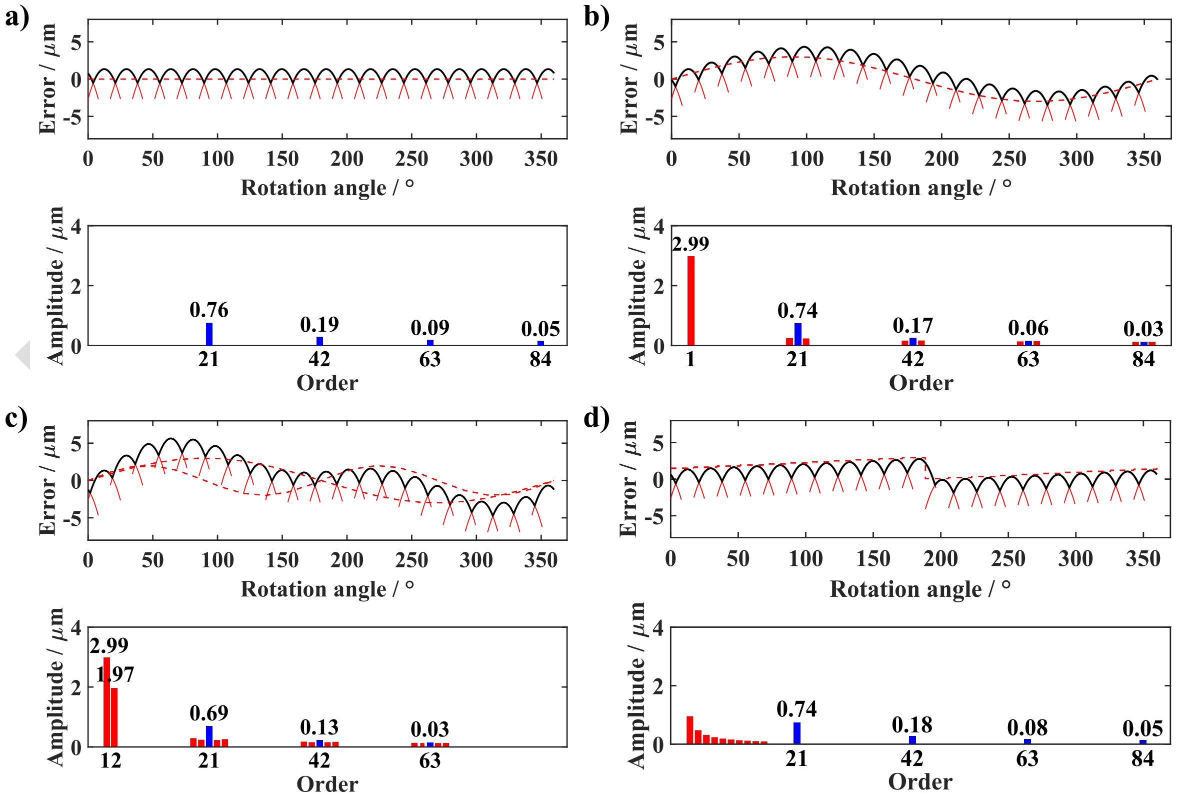


Fig. 6. Analysis of transmission error: a) eccentricity-free condition, b) eccentricity radial runout, c) radial runout and double strike, d) pitch deviation.

4.2. Analysis of transmission error with irregular tooth surface texture

The tooth surface texture is closely linked to noise behaviour. By improving irregular micro-geometries, such as surface waviness and roughness, significant reductions in noise and vibration at the contact tooth flanks can be achieved. Figure 7 shows the tooth surface texture with manufacturing errors, where a peak with an amplitude of 0.30μm appears at the 67th harmonic position, indicating the presence of an error component with a period of 67 in the transmission error. Analysis reveals that this component originates from the surface texture introduced during tooth flanks processing. It is the irregularities in the tooth surface texture, resulting from the processing, that give rise to ghost orders in the spectrum. These ghost orders primarily contribute to the whine noise of the gear during meshing.

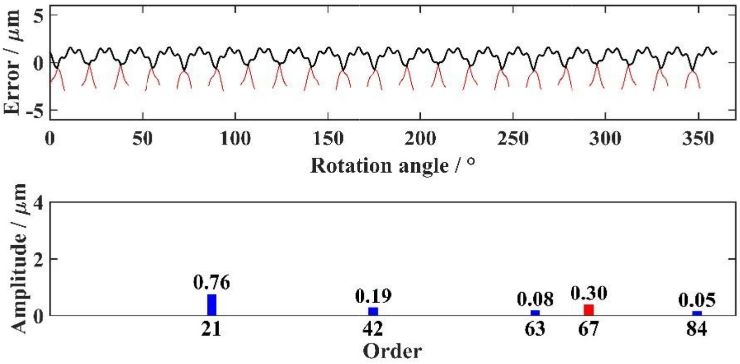


Fig. 7. Analysis of transmission error containing ghost orders.

5. Experimental analysis

Figure 8 shows the Klingelnberg P26 gear measuring center that was used to measure the measured gear. The measurement items included key gear geometric accuracy parameters such as profile deviation, helix deviation, and cumulative pitch deviation, which were quantitatively evaluated according to ISO 1328-1:2013 standards. The instrument's integrated multi-axis probing system and advanced evaluation software ensured traceable measurement uncertainty below 0.5μm. The gear measurement parameters are shown in Table 4 below.

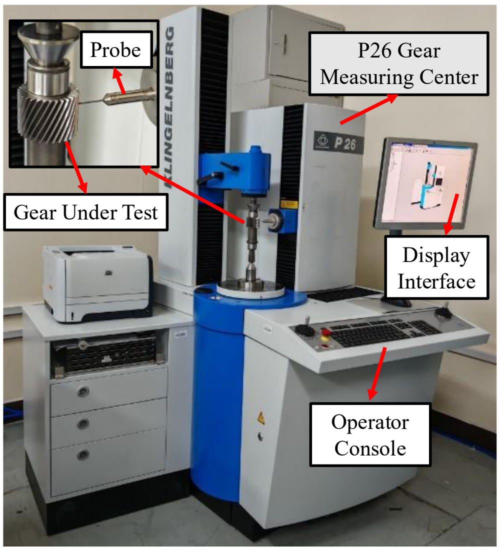


Fig. 8: Measurement on the gear measuring center (Klingelnberg P26).

Table 4. Examples of renewable journals from the field of measurements and instrumentation.

Number of Teeth	**Module/mm**	**Pressure angle/°**	**Helix angle/°**	**Face width/mm**
21	1.658	20	23.2	32

5.1. Analysis of transmission error with irregular tooth surface texture

So far, the noise analysis from gear deviations has not considered the effect of corner contact on gear transmission error. Actually, the effect of corner contact on gear transmission error is apparent, taking gear with negative pressure angle deviation as an example, as shown in Fig. 9. The comparison is shown in Fig. 10. When corner contact is considered, the amplitude of the first meshing order is large but that of other meshing orders is smaller. Therefore, in transmission error analysis, the corner contact should not be overlooked.

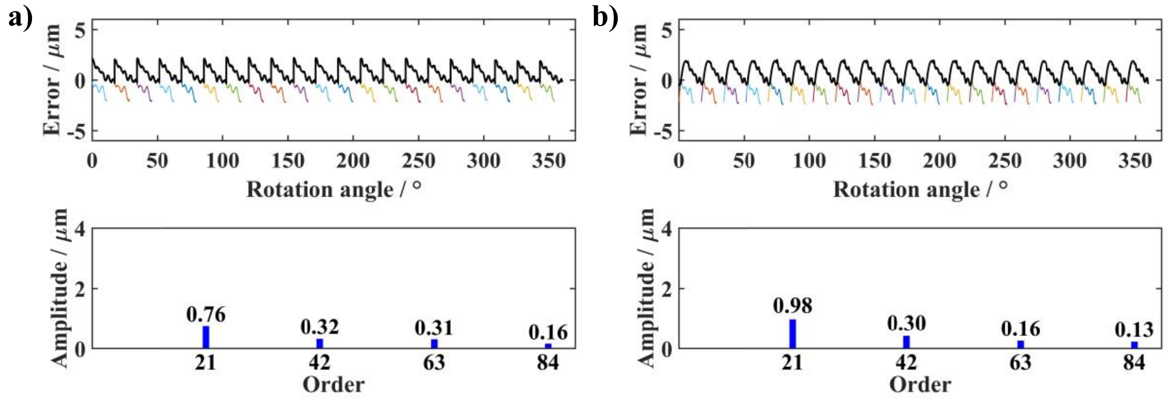


Fig. 9. Comparison analysis: a) no considering corner contact, b) considering corner contact.

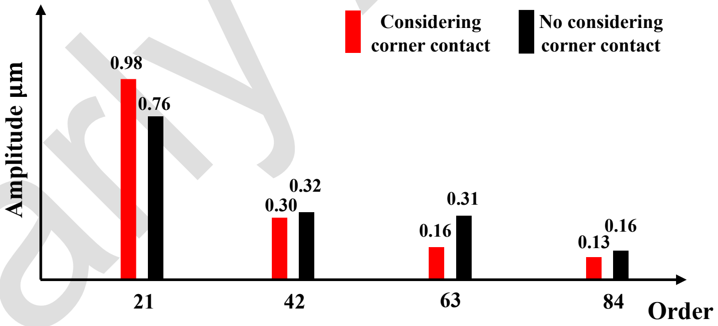


Fig. 10. Comparison analysis with no considering corner contact and considering corner contact.

5.2. Effect of pressure angle deviation on gear transmission error

Figure 11a shows the analysis results at a 20-degree pressure angle. From the analysis of Fig. 9b with negative pressure angle deviations and Fig. 11b with positive pressure angle deviations, it can be observed that the spectral results remain identical if the absolute values of the positive and negative pressure angle deviations are equal. Actually, meshing-in impact occurs in gears with negative pressure angle deviation, while meshing-out impact occurs in gears with positive pressure angle deviation [25]. The noise of the former is much greater than that of the latter. Therefore, there is a possibility that inaccurate noise prediction may occur in spectral analysis based on gear deviation.

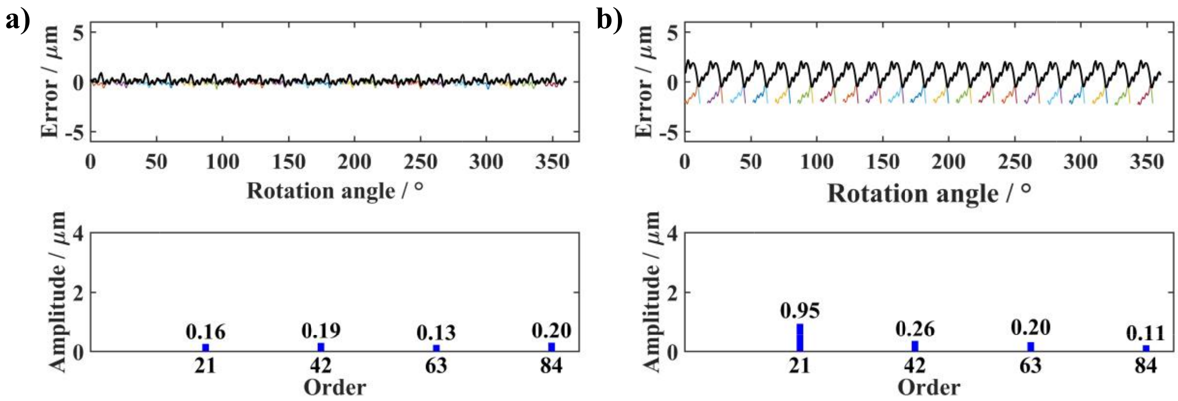


Fig. 11: Comparison results: a) standard gear, b) gear with positive pressure angle deviation.

5.3. Ghost orders in gear transmission analysis

In practical gear transmission systems, ghost orders refer to spurious frequency components that are not directly related to the gear meshing process. In the measurement results shown in Figure 12, ghost orders were identified at the 26th and 47th orders, each exhibiting an amplitude of approximately 0.05µm. Although their amplitudes are relatively small, these ghost orders can still have a significant impact on gear whine noise by introducing non-physical excitations into the system. Such spurious components may excite system resonances or interfere with the natural meshing harmonics, thereby amplifying tonal noise.

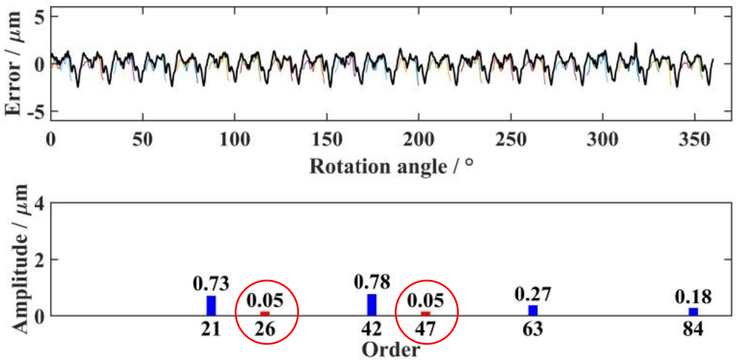


Fig. 12. Ghost order noise-causing gear whine noise.

5.4. Guidelines for industrial application

In industrial practice, the six typical gear deviation shapes and their corresponding tooth-pair change-over modes should be incorporated into routine inspection standards. Priority should be given to the measurement of corner contact-related parameters, and the proposed least-squares sine wave fitting method should be adopted to quantify the deviations of non-drum gears. For gear noise fault diagnosis, both time-domain and frequency-spectrum analyses should be combined. When ghost orders are detected, the machining quality of the tooth surface should be improved. During the manufacturing process, targeted control should be implemented on key gear deviation sources—for instance, eccentricity error can be reduced through fixture calibration, and pitch deviation can be alleviated by dressing grinding wheels. Priority should be given to the correction of negative pressure angle deviations. The corner contact effect should be integrated into gear design, and a parabolic transmission error model should be employed to optimize the tooth profile, so as to reduce vibration and noise in high-precision applications such as transmissions for new energy vehicles.

6. Conclusions

As the pivotal transmission components, gears directly determine the operational stability of mechanical systems. In the field of electric vehicles subject to stringent noise control specifications, the control of gear transmission noise is particularly critical. To tackle the problem of gear noise at its root cause, this study analyzes gear deviations, corner contact meshing, and transmission errors of gear pairs, and the main conclusions are drawn as follows:

1) A novel least-squares sine wave fitting method is proposed to separate gear deviations. It is found that the parabolic surrogate model of corner contact increases the amplitude of the first-order meshing frequency spectrum while reducing the amplitudes of other order components, revealing that the effect of corner contact cannot be neglected in transmission error analysis.
2) Six typical gear deviation shapes and six gear pair tooth-pair change-over modes are identified, and the gear transmission errors under different modes are analyzed, with special attention paid to the key influencing factor, *i.e.*, corner contact.
3) For gears with significantly distinct noise characteristics caused by positive and negative pressure angle deviations, the spectral analysis results exhibit similarity, indicating that relying solely on spectrum analysis for noise-causing gear fault diagnosis has inherent limitations.

Acknowledgements

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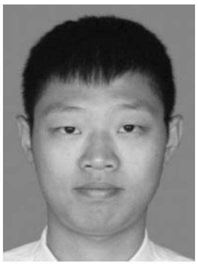

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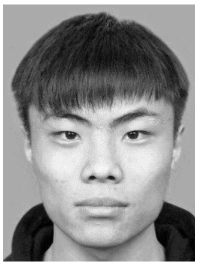

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