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SIGNAL DENOISING USING A LOW COMPUTATIONAL TRANSLATION-INVARIANT-LIKE STRATEGY INVOLVING MULTIPLE WAVELET BASES: APPLICATION TO SYNTHETIC AND ECG SIGNALS

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Abstract

In this paper, an efficient method for the denoising of electrocardiogram (ECG) signals is presented. As it is well-known, the efficient translation-invariant (TI) denoising technique, first introduced by Coifman and Donoho, uses K preprocessing shift-rotation operations, K denoising operations similar to the standard Donoho's thresholding algorithm, K post-processing inverse shift-rotation operations and finally, the K new less noisy generated copies by the preceding steps are averaged to produce a final denoised signal. Thus and conversely to the previously mentioned TI algorithm, the suggested technique consists of the design of a low computational translation-invariant-like strategy that eliminates the K preprocessing shift-rotation and the K post-processing inverse shift-rotation operations and only keeps the K wavelet-based denoising operations where for each one, we use a different mother wave among a set of K mother waves $\psi_1, \psi_2, \ldots, \psi_K$. Consequently, each mother wave generates a new less noisy copy from the original noisy signal. Finally, the produced less noisy multiple copies are averaged to reach the final denoised signal. Through this strategy, we can avoid the use of multiple hardware sensors to generate multiple noisy copies to be averaged to restore the clean version of the signal. Consequently, the proposed approach can considerably reduce the cost of the acquisition system. Additionally, the several results produced from extensive achieved simulations show that the proposed algorithm outperforms many translation-invariant-like methods and can be considered as one of the top-ranked recent algorithms treating the denoising problem.

Keywords: ECG signals, set of wavelets, translation-invariant, Donoho's denoising, noisy copies generation from a single record, white gaussian noise.

1. Introduction

Nowadays, healthcare systems that diagnose and detect human pathologies play a significant role in our modern daily individual lives. Such systems must be efficient and accurate to process and analyze several bio-signals (ECG, EEG, EMG, PCG, *etc.*) in a manner that avoids false analysis and wrong judgment. Among the problems that can be encountered in the design of health monitoring tools is the noise or disturbance acting in data acquisition phase and/or in distant transmissions. Consequently, the ECG signal, which is the electrical activity of the human heart, can be noised by many sources (power line interference, contact noise, patient–electrode motion artifacts, EMG noise, baseline drift, *etc.*) [1]. To correctly decide about the cardiac health status, the noise must be, as much as possible, efficiently reduced. In this context, several contributions have been accomplished. One of the noise types commonly treated is the *additive white Gaussian noise* (AWGN). We can categorize, essentially, previous works relating to the subject as follows:

– Techniques derived from image processing, such as morphological filters. Such methods use (erosion, dilation, opening, and closing) [2-4].

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- Strategies based on ICA and/or PCA, as cited in [5-7].
- Filtering algorithms such as the method using an adaptive filtering system [8-12].
- Methods using the recent transform known as *Empirical Mode Decomposition* (EMD) [13-15].
- Reported schemes based on the wavelet transform, such as those mentioned in [16, 17].
- Combination-based methodologies merging as examples: Wiener/Kalman filters [18], wavelet/Savitzky-Golay filter [19], wavelet/Wiener filters [20], wavelet/fuzzy reasoning [21].

However, we attract the reader's attention to the algorithms based on the special wavelet-thresholding case, which can be considered as variants of the standard state-of-the-art (Donoho's) algorithm [22, 23]. In [24], the authors presented a method based on nonlinear thresholding of wavelets and optimized wavelet packet coefficients using hard and soft thresholding, investigating four well-known strategies (Sure, Heuristic Sure, Fixthres, and Minimax). As reported, the results of wavelet denoising methodologies showed superior performance than wavelet packets for white noise. Also, in [25], a scheme based on second-generation (Lifting scheme) wavelets with level-dependent thresholds determination was suggested. The authors reported that the obtained results are depending on the wavelet type filters, the applied thresholding method, and the decomposition depth. The method has superior performance when faced with the median filter and is faster than traditional wavelet decomposition. Another wavelet-based methodology is given in [26]. It proffers, mainly, a wavelet-based scheme to denoise corrupted ECG records by using conventional wavelet soft-thresholding (shrinkage) using a level of decomposition 8 (selected empirically) and the Daubechies wavelet Db4 (of 4 vanishing moments). In the same optic, the contribution shown in [27] consists of recovering a clean ECG signal from its noisy version using a subband-dependent threshold. The so-called S-median thresholds calculated by this technique can be considered as a variant of the well-known Donoho's universal threshold. Moreover, in [28], the presented contribution is based on the genetic algorithms optimization of several parameters that allow the successful cleaning of noisy ECG. These parameters are (as mentioned): the mother wave, the decomposition level, the type and the rule of the threshold, and finally the rescaling approach. Another denoising strategy, developed by Y. Yang *et al.* [29], consists of reducing white Gaussian noise and particularly suppressing the pseudo-Gibbs phenomena with universal threshold by the *Random interpolation average* (RIA) technique. In this latter, independent denoised signals were obtained by each time interpolation and denoising. The pseudo-Gibbs phenomenon was suppressed by calculating the mean of all the independent denoised signals. In [30], the signal denoising method was based on Translation-invariant, DWT and *goodness-of-fit* (GOF) tests. This approach performs on several scales; it consists of determining which DWT coefficients represent noise and removing them by using GOF statistical examinations. Additionally, Naveed *et al.* [31] adopted a strategy that is based on the combination of statistical neighbourhood dependencies of DWT coefficients and GOF test. This allows classifying coefficients as signal or as Gaussian noise. Also, in [32], Talbi made a strategy applied to ECG signals based on the one-dimensional double-density complex DWT noise reduction technique in the *stationary bionic wavelet transform* (SBWT) domain. Furthermore, Zhang *et al.* [33] presented a denoising of ECG signals contaminated by white noise. This method is made by combining wavelet energy with a smoothing filter. Moreover, Liu et al [34] proposed an ECG denoising approach based on the *basis pursuit* technique (BP) and *alternating direction method of multipliers* (ADMM) optimization. In this recent work, the combination of low-pass filtering and compressed sensing recovery achieved a high signal-to-noise ratio.

Based on the problem statement and reported literature survey, we present in this work an improved version of the standard state-of-the-art Translation-invariant method [35] showing superior performances compared to some powerful recent contributions. The main idea of the proposed scheme is the use of a set of mother waves in the standard Donoho's algorithm and it is noted that each mother wave gives a resulting less noisy ECG record. Consequently, it will be shown, by results that averaging the resulting less noisy ECG signals will significantly improve the recovery process.

The paper is organized according to: Section 2 which presents the standard algorithm of Donoho [22, 23]. In Section 3, the Translation-invariant technique is resumed. In Section 4, the proposed improved methodology is described in detail. Results followed by thorough discussions, are given in Sections 5 and 6 respectively. Finally, concluding main results and remarks with possible future works are summarized in Section 7.

2. Donoho's Standard wavelet-thresholding-based algorithm and other reference denoising methods

The conventional statement of the problem of denoising a noisy signal contaminated by an *additive white Gaussian noise* (AWGN) is usually given by:

$$Ns(i) = Fs(i) + n(i) \qquad / \qquad i = 0,1,\, ...,\, L\text{-}1, \tag{1}$$

where:
Ns : is the noisy signal of length L,
Fs : is the unknown free-noise deterministic signal of length L, to be recovered (estimated),
n : is the L samples of an i.i.d. (independent and identically distributed) white Gaussian noise that follows the probability density function $N(0,\sigma^2)$ [22]. $N(0,\sigma^2)$ is the normal distribution of zero mean and standard deviation σ.

The Donoho's Method [22] for denoising can be summarized as follows:
- Application, on the noisy signal, of the *discrete wavelet transform* (DWT) was introduced by Mallat in [36].
- Thresholding of wavelet coefficients.
- Recovery of the estimated unknown signal by applying the inverse DWT.

The two well-known thresholding strategies that were proposed by Donoho *et al.* [22] are presented by the following:

- Soft thresholding strategy:

$$\mathrm{WCTH}(i) = \begin{cases} \mathrm{WC}(i) - \mathrm{TH} & \mathrm{WC}(i) \geq \mathrm{TH}. \\ 0 & |\mathrm{WC}(i)| < \mathrm{TH}. \\ \mathrm{WC}(i) + \mathrm{TH} & \mathrm{WC}(i) \leq -\mathrm{TH}. \end{cases} \tag{2}$$

$\mathrm{WC}$is the wavelet coefficients vector of the noisy signal Ns. WCTH is the thresholded wavelet coefficients vector resulting from the thresholding process using the threshold TH.

- Hard thresholding strategy:

$$\mathrm{WCTH}(i) = \begin{cases} \mathrm{WC}(i) & |\mathrm{WC}(i)| \geq \mathrm{TH}. \\ 0 & |\mathrm{WC}(i)| < \mathrm{TH}. \end{cases} \tag{3}$$

Several methods for the choice of the threshold(s) have been suggested in the specialized literature. The well-known standard universal threshold [22] is described by:

$$\mathrm{TH} = \sigma\sqrt{2\log(L)}. \tag{4}$$

Also, we can mention other methods based on the minimax estimation principle [23].

For practical use in the real world, σ is actually unknown and should be appropriately estimated. A valid estimation is shown in the following equation [22, 23]:

$$\sigma = \frac{\mathrm{MAD}(\mathrm{CD}_1)}{0.6745}. \tag{5}$$

Where: M.A.D. is the abbreviation of Median Absolute Deviation and CD_1 are the fine-scale details wavelet coefficients.

In the reported methods, we can see two thresholding rules: the first one is the global or fixed threshold rule and the second one is the level-dependent threshold strategy.

3. Overview of Translation-invariant denoising principle

Translation-invariant "TI" denoising is a technique used to address the issue of translation dependency in traditional wavelet-based denoising methods that incorporate the down-sampling (decimation) and the up-sampling operators. In the traditional wavelet transform, the basic functions are not invariant to translations, which can result in artifacts near discontinuities. Translation-invariant denoising methods can better preserve the details and edges of the signal while effectively reducing noise. Translation-invariant denoising methods can be computationally more demanding compared to traditional wavelet-based methods.

Visual artifacts, such as Gibbs phenomena near discontinuities, can indeed be a result of the lack of translation invariance in the wavelet basis. One method to address this and reduce such artifacts is called "cycle spinning," a term coined by Coifman *et al.* [35]. The idea behind cycle spinning is to "average out" the translation dependence by iteratively applying translations to the signal and computing the average of the transformed signals. By performing this iterative translation and averaging process, cycle spinning aims to mitigate the translation dependency and reduce visual artifacts in the denoised signal.

The TI technique involves applying a range of shift-rotation operations to a signal and then averaging the results to produce a reconstruction with reduced noise phenomena. Averaging the results from different shift-rotation operations allows a more robust and accurate reconstruction.

In Fig. 1, the denoised signal $\hat{x}(n)$, involving the use of the conventional Translation-invariant strategy, is obtained by using four steps. In the first step, a range of all circulant right shifts is applied to the noised signal $x(n)$. Donoho's denoising with a fixed one-wavelet mother ψ is used in the second step. Next, a range of all circulant right shifts is applied. Finally, an average of the obtained results is calculated to reconstruct $\hat{x}(n)$.

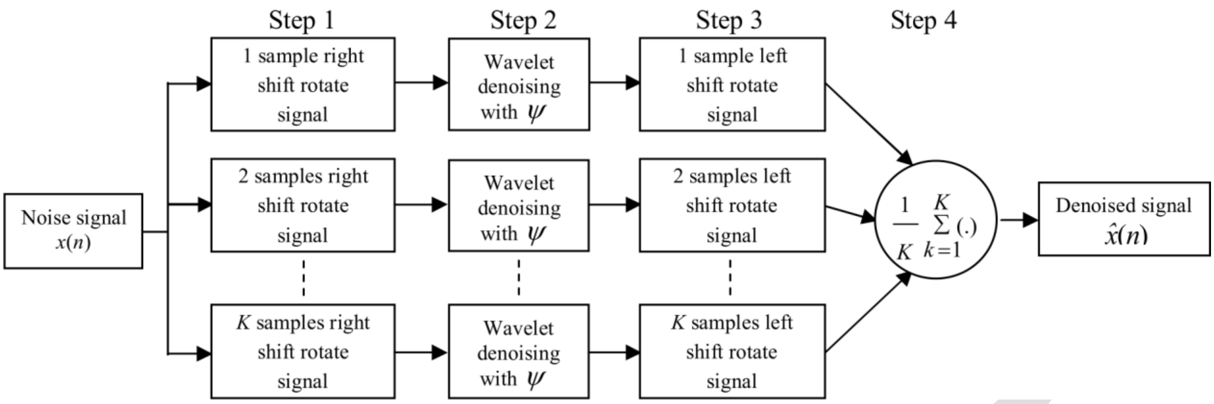

Fig. 1. Principle of the Translation-invariant technique.

4. Proposed low computational method description

The proposed method is based, essentially, on the work of Coifman *et al.* [35] "Translation-invariant" which is well-known as the universal threshold with cycle spinning, in which we use the hard universal thresholding previously mentioned. The main step used in the suggested method is inspired by the well-known noise cancellation problem in the case of multi-sensor data fusion. The rightful question that one can ask is: How can we obtain from a single noisy data signal multiple copies? Equivalently, in other words, how can we reproduce a multi-sensor environment from a real single sensor? The answer is the generation, from the original single signal record, of different noisy signals by using the standard Donoho's noise reducer algorithm, and finally, in order to improve noise cancellation, the data-fusion principle is used.

4.1. Multiple copies generation of noisy signals from a single noisy signal

As it is well established, the universal or minimax thresholding algorithm aims to reduce the additive noise. It means that in real cases, the noise is not completely eliminated, but its effect is reduced. The key step of our suggested variant of the Translation-invariant standard algorithm is the use of K several mother waves to generate K new noisy signals under the assumption that each newly generated noisy record contains a deterministic unknown noise-free signal and a realization of new additive i.i.d. noise produced by the same new process. Note that in the TI algorithm; just one mother wave with two ranges of shift rotations is used. With our proposed algorithm, a low computational strategy is applied without any shift or rotation.

Let $\psi_1, \psi_2, .., \psi_K$ be K mother waves. We apply, for each one, the universal or minimax thresholding algorithm to obtain the new resulting noisy signal according to the schematic shown in Fig. 2.

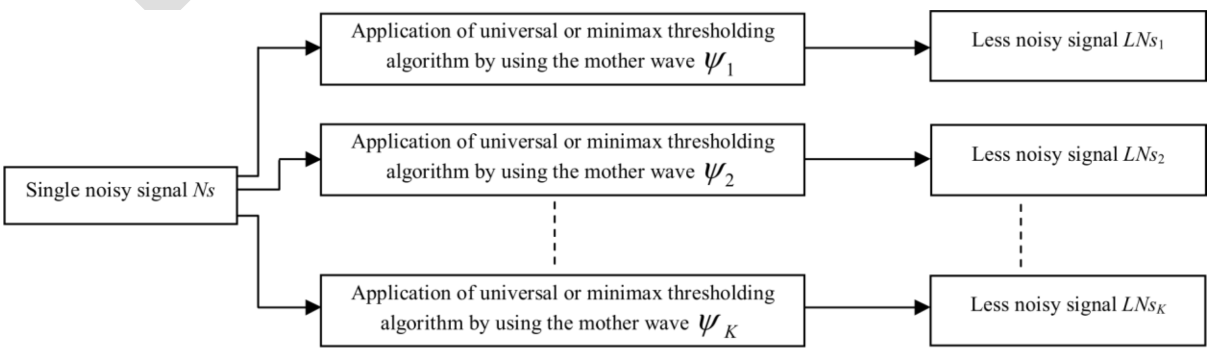

Fig. 2. Multiple-copies Noisy signal generation from an original single-sensored noisy signal.

Each k^{th} less noisy signal LNs_k / k=1, 2, ..., K, is modelled following (6):

$$LNs_k = Fs + Nn_k, \tag{6}$$

where:

LNs_k : is the k^{th} less noisy generated signal, of length L, by the use of the mother wave ψ_k in the universal or minimax thresholding algorithm,

Fs : is, as mentioned to describe (1), the unknown free-noise deterministic signal, of length L, to be recovered (estimated),

Nn_k : is the k^{th}new realization of a vector noise signal (process Nn) of length L. The new random process Nn is assumed to be i.i.d. zero mean process. It means that each i^{th} sample of the noise process is zero mean ($E(Nn(i)) = \mu_{Nn}(i) \to 0$).

Compared to Translation-invariant, our proposed technique eliminates all the spinning cycles and uses multiwavelet thresholding.

4.2. Improvement of the noise cancellation by data fusion principle

The recovered estimated signal using the averaging technique (sample by sample) approaches the most possible free-noise deterministic unknown signal in respect of the following equation:

$$E(LNs(i)) = E(Fs(i) + E(Nn(i)) \quad / \quad i = 0,\ 1,\ ...,\ L-1. \tag{7}$$

The i^{th} resulting estimated sample, let be denoted $\hat{Fs}(i)$, is then described by simplifying (7):

$$Fs\ \ i\ \ = Fs\ \ i\ \ + \mu_{Nn}\ \ i\ \ , \tag{8}$$

where: $\mu_{Nn}(i) \to 0$ (approaches 0 the most possible when using K mother waves than the use of the single best mother wave) leading to the result:

$$\hat{Fs}(i) \cong Fs(i). \tag{9}$$

Note that the expectation operator $E(.)$ can be calculated by:

$$E(.) = \frac{1}{K}\sum_{k=1}^{K}(.). \tag{10}$$

This well-known technique, usually used in the data-fusion discipline [37] can be described in Fig. 3.

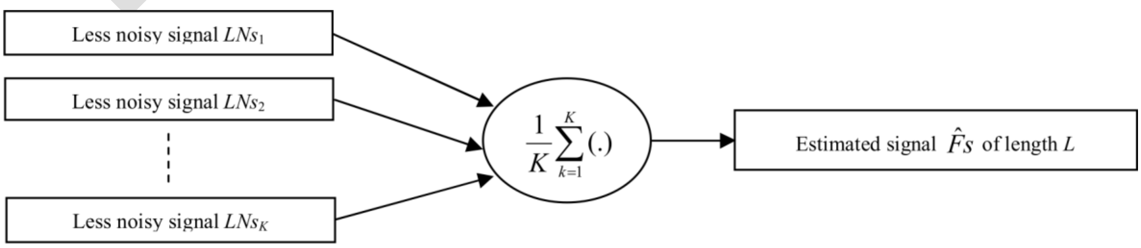

Fig. 3. Noise reduction using the average operator to fuse (sample by sample) the K less noisy signals (LNs_1, LNs_2, ..., LNs_K).

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5. Results

The proposed technique was evaluated using quantitative criteria. The parameters used are SNR_in (input signal to noise ratio) in dB, SNR_out (output signal to noise ratio) in dB and MSE (Mean Square Error). They are given respectively by (11), (12) and (13).

$$\mathrm{SNR\_in} = 10\log_{10}\left(\frac{\sigma^2_{ori}}{\sigma^2_{noise}}\right) \tag{11}$$

$$\mathrm{SNR\_out} = 10\log_{10}\left(\frac{\sigma^2_{ori}}{\sigma^2_{ori-denoised}}\right) \tag{12}$$

where:

σ^2_{ori} : represents the variance of the original signal,

σ^2_{noise} : is the noise variance,

$\sigma^2_{ori-denoised}$ *:* denotes the variance of the difference between the clean signal (original) and the denoised one.

$$\mathrm{MSE} = \frac{1}{L}\sum_{n=1}^{N-1}\left(x\ n\ - x\ n\ \right)^2. \tag{13}$$

where *x*(*n*) and $x\ n$ are the samples *n* of the clean and the denoised signals (of lengths *L*), respectively.

5.1. Results relative to synthetic signals

In the first step of our simulations, our approach is evaluated by using four well-known test signals named 'Blocks', 'Bumps', 'Heavy Sine' and 'Doppler'. We have chosen several lengths ranging between 128 and 8192 samples. The considered corrupting noise is the additive white Gaussian noise with a zero mean and a variance that depends on the SNR_in. The shapes of noised signals, for SNR_in equal to 10 dB and with a length of 8192 samples, are shown in Fig. 4.

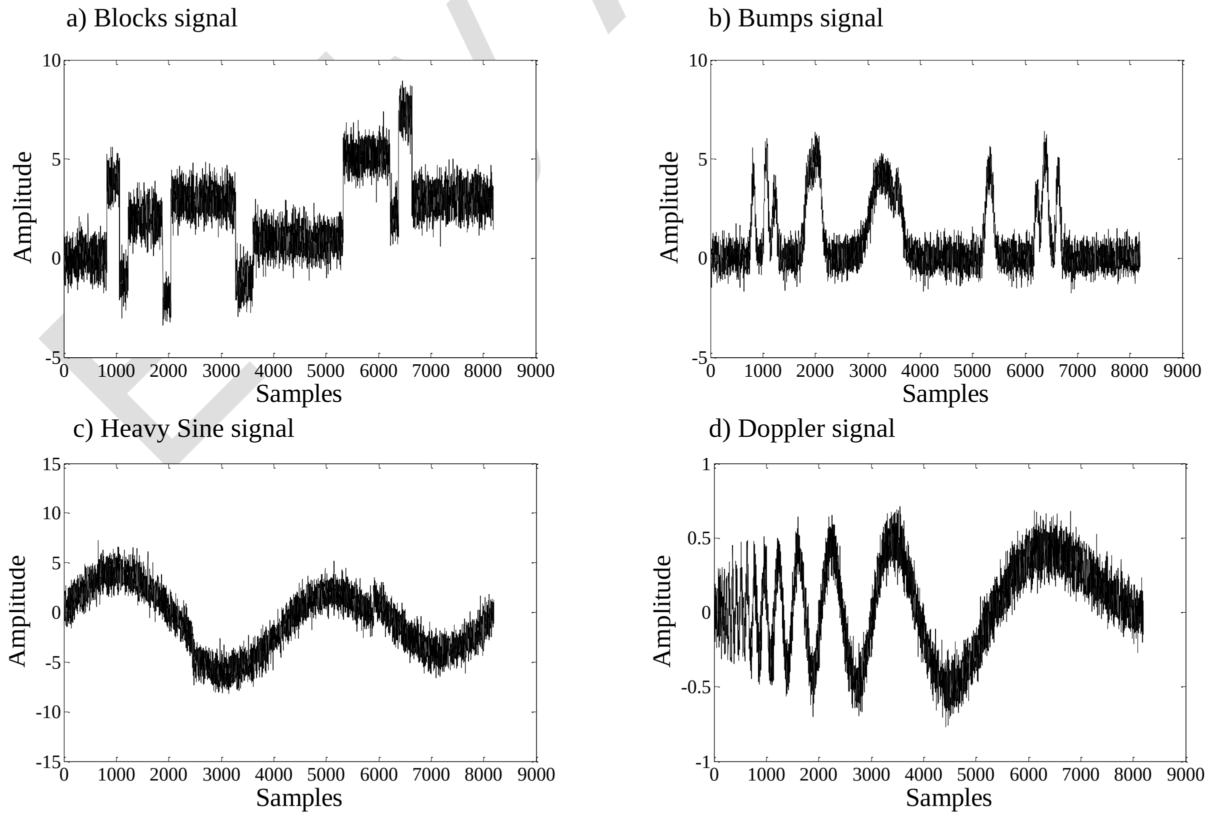

Fig. 4. Shapes of noised signals for signals length=8192 samples and SNR_in=10 dB.

For the denoising process, a set of 21 wavelet mothers {Db1,.., Db8, Coif1,..., Coif5, Sym1,…, Sym8} is used. The employed parameters are the hard universal threshold and wavelet decomposition level 7.

For the aim of the visual inspection, the waveforms of the denoised signals recovered from the contaminated ones in Fig. 4 are shown in Fig. 5. Therefore, excellent visual quality is noticed for all signals.

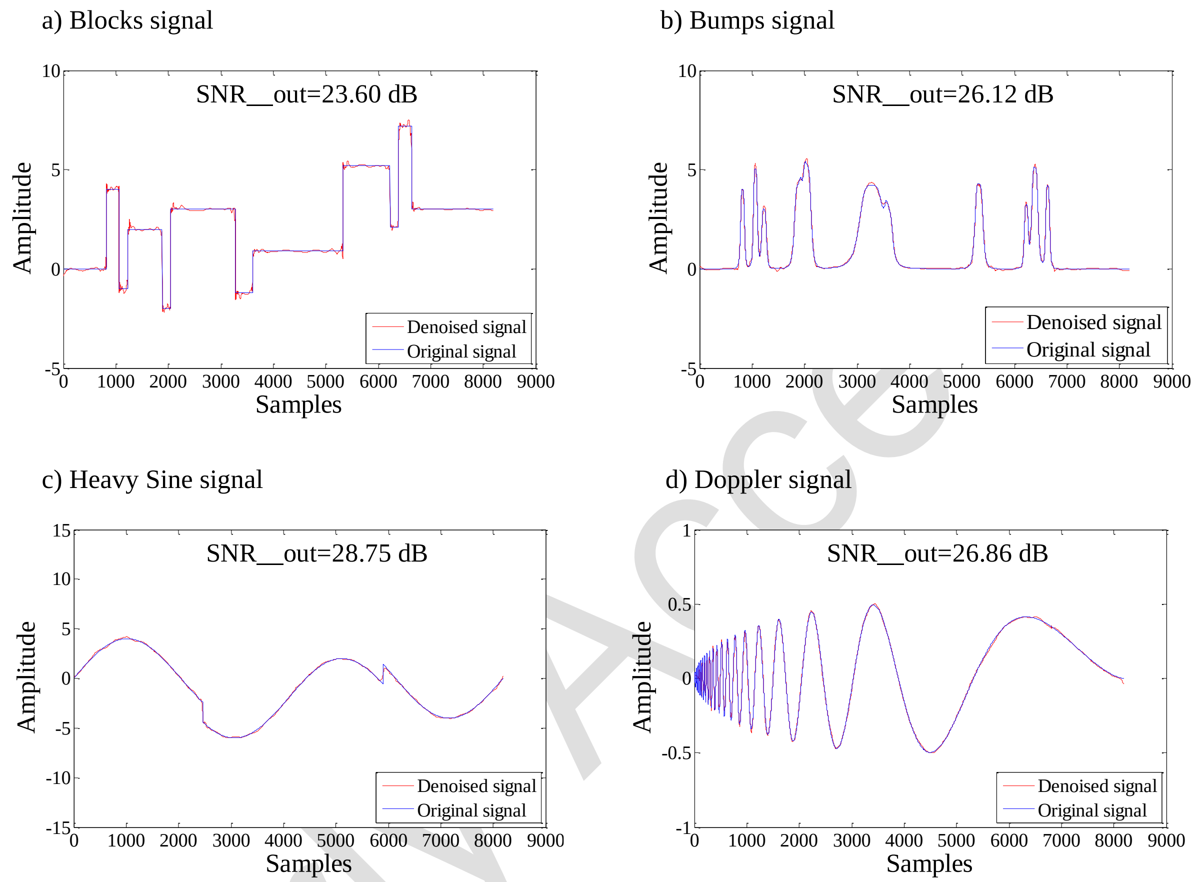

Fig. 5. Original and denoised signals for SNR_in=10 dB and signals length= 8192 samples.

5.2. Results relative to ECG signals

Our approach was also applied and evaluated for real ECG signals. These records were obtained from MIT-BIH Arrhythmia Database [38]. This latter includes a set of 48 types of two-channel ECG recordings, studied by the BIH Arrhythmia Laboratory. The duration of each data is about 30 minutes. All ECG signals were made over a 10mV range at 360 samples per second with an 11-bit resolution. All types of ECG signals from the Arrhythmia Database were used in our tests.

Note that, in this study, an additive white Gaussian noise is added to all test signals. Also, for both Donoho’s standard algorithm and the proposed one, the parameters used are the hard minimax threshold and wavelet decomposition level 4.

Our suggested denoising technique involves 4 sets of wavelets, 3 sets that are: the Set1 {Db1,..., Db8}, the Set2 {Coif1,..., Coif8}, the Set3 {Sym1,..., Sym8} and the set4 {Db1,.., Db8, Coif1,..., Coif5, Sym1,…, Sym8} constituted from all wavelets belonging to the 3 previously mentioned sets.

In Fig. 6, each LNs_k for k=1, ..., 21 (21 wavelet mothers) are generated by using wavelet denoising involving wavelet mothers of Set4. It means that LNs_1 is obtained by employing Db1

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wavelet mother, LNs_2 is produced by implying Db2, ... and finally, LNs_{21} is achieved by using Sym8. The estimated sample $\hat{F}s(i)$ is the i^{th} expectation sample $E(LNs(i))$ / $i = 0, 1, ..., L\text{-}1$.

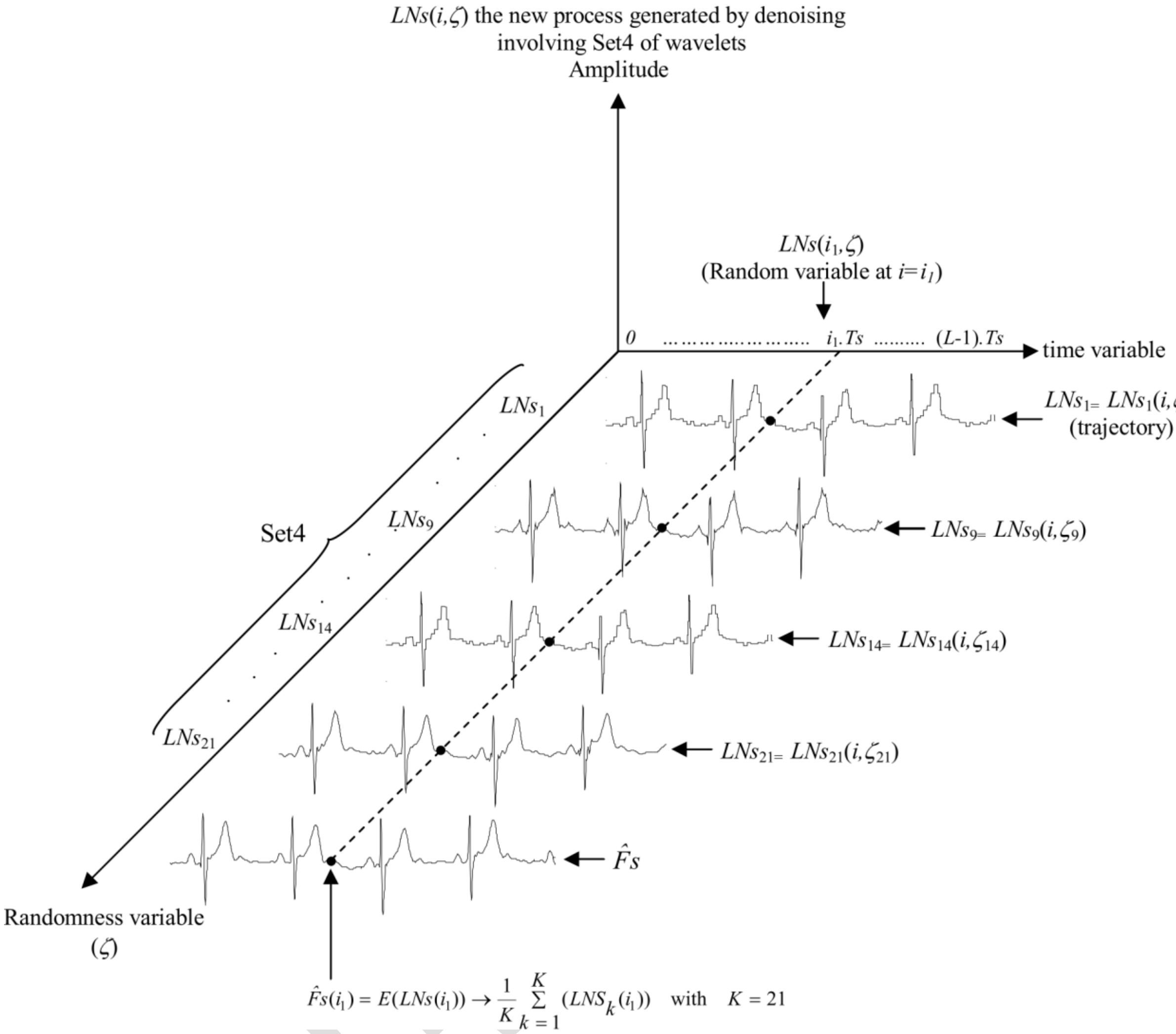
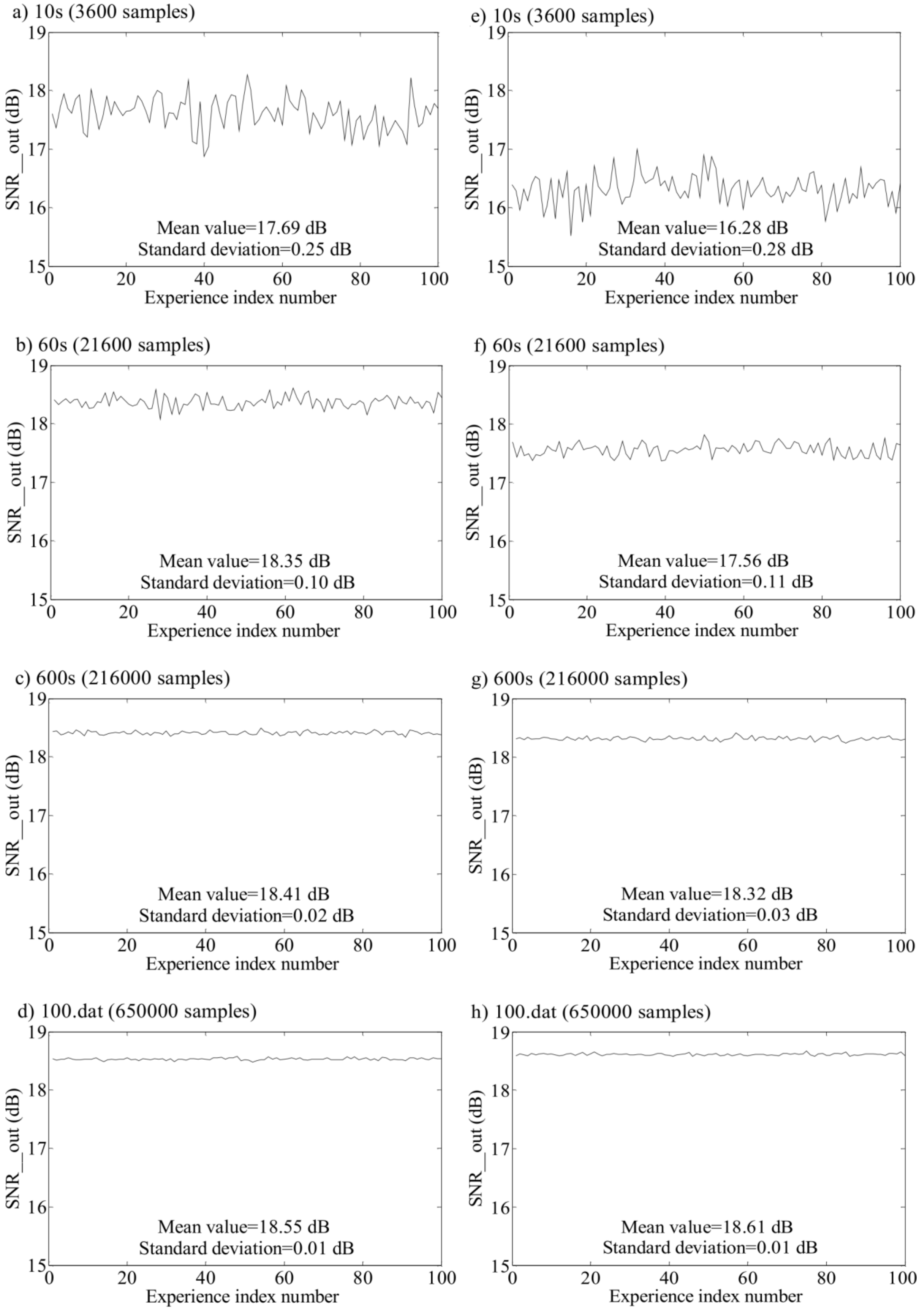


Fig. 6. Illustrative ECG denoising by the suggested strategy implying Set4.

As it is well known, when we use a small sample of a signal to confirm a developed denoising strategy for example, one can repeat the experience many times and take recognizable statistical measures such as the mean and standard deviation because of the variability of the obtained results. However, when the sample (signal) is of a large size, the variability will be significantly reduced to tend to the theoretically probabilistic case. Additionally, as it can be seen that for both methods, the standard deviation is of reduced values ranging from 0.25 dB for a small size of 10s to 0.01 dB for the entire size of the signal (See the illustrative Fig. 7). Also, the standard deviation will be much smaller as the size of the signal is longer. The given remark holds for both TI and proposed method. Accordingly, our case used signals are of a considerable length of 650000 samples, which gives, each executed experience, practically the same result.



(1) Obtained results by the proposed method (2) Obtained results by Translation-invariant

Fig. 7. SNR_out fluctuations according to the length of the signal 100.dat fo SNR_in=10 dB.

6. Discussions

6.1. Discussions related to synthetic signals

Table 1 summarizes the achieved comparison results provided when facing our technique against *Translation-invariant* (TI) [35] and RIA (random interpolation average) algorithms [29]. Consequently, one can report that SNR_out depends on the length of the test signal. Accordingly, in this table, the used lengths for each test signal are 128, 256, 512, 1024, 2048, 4096, and 8192 samples.

Note that the mean value of the SNR_out of the RIA technique is better than that obtained with TI and our approach for Blocks and Bumps. Thus, the mean values, in these cases, for RIA are equal to 18.57 dB and 19.39 dB for Blocks and Bumps respectively, in opposition to 17.86 dB and 19.29 dB provided by our technique. On the other hand, when using our approach, the obtained average values of SNR_out are 23.40 dB and 20.34 dB for Heavy Sine and Doppler signals respectively, against 23.10 dB and 19.06 dB for TI and 22.57 dB and 19.71 dB for RIA algorithm. The global mean value for all synthetic signals calculated by our approach is 20.22 dB, versus 19.02 dB and 20.06 dB for TI and RIA, respectively. Therefore, this result shows that our approach outperforms TI and RIA techniques.

Table 1. SNR_out of TI, RIA and our approach for SNR_in of 10 dB for different signal lengths.

Signal length	**Blocks**			**Bumps**		
	TI [35]	**RIA [29]**	**Our approach**	**TI [35]**	**RIA [29]**	**Our approach**
128	10.97	**12.84**	12.42	9.35	12.09	**13.16**
256	11.98	**14.72**	13.75	10.85	13.15	**13.93**
512	14.10	**16.22**	15.52	14.95	**17.86**	16.36
1024	16.30	**18.57**	17.95	17.67	19.36	**19.43**
2048	17.79	**20.46**	19.97	20.26	**22.71**	21.93
4096	20.60	**22.66**	21.87	23.77	**24.57**	24.12
8192	23.21	**24.53**	23.60	25.73	26.05	**26.12**
Mean value of SNR_out	16.42	**18.57**	17.86	17.51	**19.39**	19.29
Signal length	**Heavy Sine**			**Doppler**		
	TI [35]	**RIA [29]**	**Our approach**	**TI [35]**	**RIA [29]**	**Our approach**
128	16.86	14.89	**18.21**	10.54	10.71	**12.23**
256	**20.15**	18.78	18.42	13.71	14.61	**16.03**
512	21.22	21.30	**22.00**	16.85	**19.08**	18.50
1024	24.19	**24.60**	24.02	20.36	**21.11**	20.78
2048	25.40	25.63	**25.82**	22.95	23.04	**23.16**
4096	**26.82**	26.25	26.61	23.46	23.67	**24.82**
8192	27.10	26.57	**28.75**	25.58	25.76	**26.86**
Mean value of SNR_out	23.10	22.57	**23.40**	19.06	19.71	**20.34**

Comparative results were also carried out by using two recently published techniques that are a Translation-invariant version named TI-DWT-GOF [30] and DTCWT-GOF-NF [31]. Accordingly, the results are given in Table 2. We draw attention to the fact that the adopted length of each synthetic signal used here is 8192 with SNR_in=10 dB. Hence, for the only case of Bumps signal, TI-DWT-GOF has exceeded our approach only by SNR_out around 1 dB. However, our approach significantly outperforms the cited techniques for Doppler, Heavy Sine, and Blocks signals. Additionally, it can be observed that the average SNR_out has been improved by about 3 dB better than the other techniques.

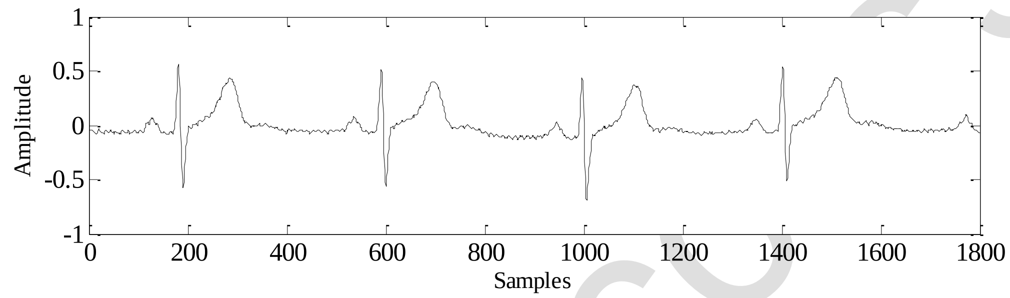
Table 2. SNR_out (dB) of denoised signals for input signals length=8192 and SNR_in=10 dB.

Techniques	Blocks	Bumps	Heavy Sine	Doppler	Average
TI-DWT-GOF [30]	19.46	**27.27**	25.46	22.92	23.77
DTCWT-GOF-NF [31]	20.25	24.93	25.02	23.45	23.41
Proposed Approach	**23.60**	26.12	**28.75**	**26.86**	**26.33**

6.2. Discussions related to ECG signals

Fig. 8a illustrates the shape of a clean 117 ECG signal and Fig. 8b shows its noisy version. However, Fig. 8c represents the restored one by using Donoho's single wavelet inimax thresholding strategy involving the mother bior2.6. However, Fig. 8d exemplifies the shape of the denoised signal by using our approach.

a)



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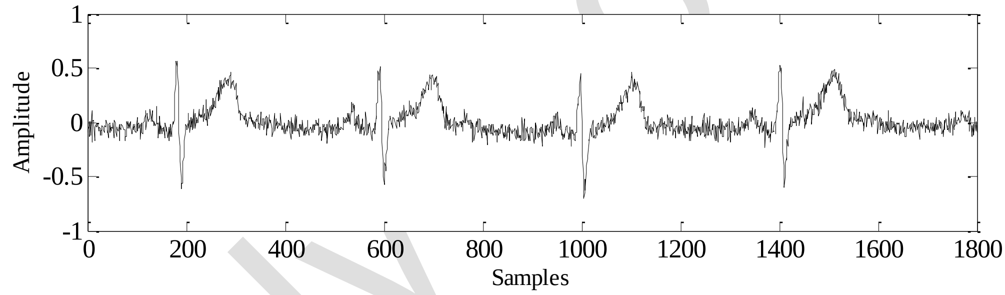
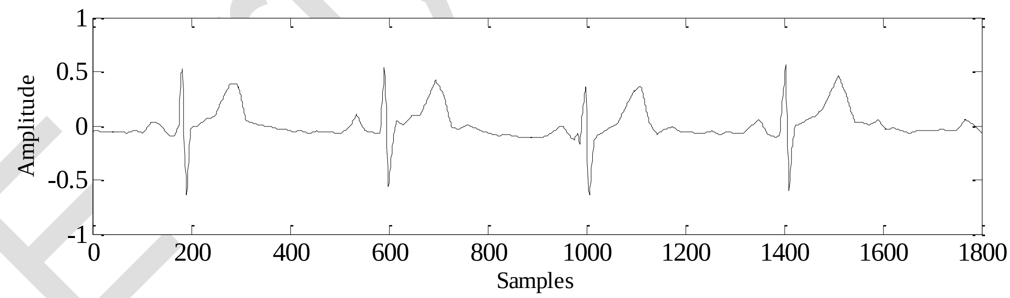


c)



d)

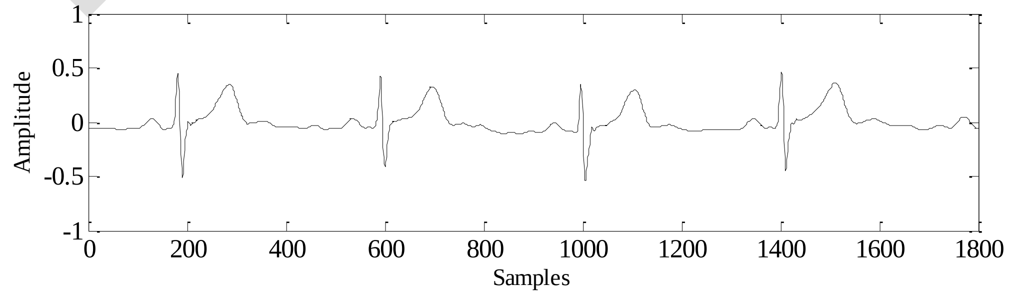


Fig. 8. Noise Cancellation of 117 ECG signal, (a) Original signal, (b) Noised signal with SNR_in=10 dB, (c) Denoising signal using Donoho's algorithm [22] (SNR_out=17.06 dB), (d) Denoising signal using our approach (SNR_out=19.17 dB).

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The SNR_in used in this experiment is fixed to 10 dB. Accordingly, the obtained SNR_out is 17.06 dB for Donoho's technique against an SNR_out of 19.17 dB reached by our strategy when using the set of eight wavelets (Sym1...Sym8). It is clear, as well, by visual inspection, that the restoration process of our approach is better than the conventional Donoho's.

For a fixed SNR_in of 10 dB, comparative results are reported in Table 3. The comparison is achieved by facing the Donoho's standard denoising approach and translation-invariant algorithm to our suggested denoising technique.

Table 3. Corresponding SNR_out (dB) for 48 MIT-BIH Arrhythmia records using SNR_in=10 dB, minimax hard threshold and wavelet decomposition level 4.

Record	**Donoho's method [22]**	**TI [35]**	**Proposed approach for a set of wavelets**			
	Single bior2.6	**Single bior2.6**	**Set1**	**Set2**	**Set3**	**Set4**
100	16.47	**18.61**	18.49	17.44	18.36	18.55
101	17.22	**19.34**	19.10	18.20	19.13	19.25
102	16.08	**18.27**	17.89	17.05	17.99	18.09
103	17.01	19.48	19.58	18.64	19.60	**19.75**
104	15.71	**18.06**	17.49	16.59	17.58	17.68
105	16.94	**19.06**	18.82	18.39	18.84	18.96
106	18.96	19.11	18.91	18.37	19.04	**19.13**
107	17.54	**19.67**	18.96	18.78	19.25	19.25
108	**19.25**	18.29	17.90	17.67	17.96	17.99
109	17.73	19.93	19.74	19.73	19.84	**19.99**
111	16.73	**18.86**	18.41	18.18	18.57	18.66
112	16.56	**18.76**	18.44	17.93	18.58	18.66
113	17.21	19.63	19.55	18.61	19.59	**19.71**
114	15.80	**17.69**	17.51	16.79	17.52	17.60
115	17.17	19.61	19.78	18.76	19.79	**19.97**
116	17.28	19.67	19.73	18.99	19.83	**19.98**
117	17.06	19.11	19.14	18.68	19.17	**19.27**
118	15.82	**17.92**	17.41	16.94	17.64	17.66
119	17.36	19.64	19.81	19.11	19.78	**19.94**
121	18.16	20.07	20.12	20.06	20.24	**20.29**
122	16.68	18.97	18.84	18.48	19.08	**19.16**
123	17.05	19.15	19.37	18.68	19.38	**19.52**
124	17.87	19.96	19.82	19.95	20.22	**20.24**
200	16.42	**18.43**	18.00	17.60	18.11	18.20
201	17.35	19.49	19.43	18.79	19.37	**19.55**
202	17.70	19.71	19.56	19.11	19.63	**19.78**
203	16.54	**18.61**	18.19	18.07	18.33	18.40
205	15.76	18.49	18.50	17.63	18.57	**18.72**
207	17.70	**19.56**	19.35	19.14	19.39	19.47
208	16.98	**19.25**	18.87	18.44	19.04	19.13
209	15.42	**17.75**	17.47	16.69	17.54	17.68
210	17.05	**19.06**	18.81	18.41	18.92	19.03
212	15.25	**17.51**	17.28	16.43	17.24	17.38
213	16.59	**19.04**	18.77	18.13	18.83	19.00
214	17.73	19.76	19.70	19.23	19.70	**19.88**
215	15.29	**17.59**	17.16	16.58	17.28	17.39
217	17.74	**19.79**	19.37	19.21	19.61	19.67
219	17.72	19.98	20.09	19.38	20.10	**20.26**
220	16.86	19.18	19.18	18.15	19.28	**19.43**
221	17.04	19.28	19.29	18.58	19.28	**19.45**
222	15.67	**17.75**	17.51	16.78	17.63	17.71
223	17.54	19.89	19.68	19.14	19.87	**19.98**
228	16.75	**18.51**	18.10	17.92	18.24	18.27
230	16.75	19.30	19.31	18.49	19.27	**19.46**
231	16.94	19.23	19.41	18.43	19.32	**19.51**
232	15.58	17.49	**17.61**	16.93	17.37	17.58
233	17.16	19.47	19.24	19.01	19.40	**19.54**
234	16.83	19.19	19.29	18.68	19.41	**19.56**
Average SNR_out	16.70	19.00	18.83	18.27	18.91	**19.02**

Accordingly, Table 3 shows that the average SNR_out of all ECG signals is 16.70 dB by using the conventional Donoho's algorithm against average SNR_out of 18.83 dB, 18.27 dB and 18.91 dB when using the proposed technique involving {Db1, ..., Db8}, {Coif1, ..., Coif5} and {Sym1, ..., Sym8} respectively. It is noticeable, that the set of wavelets {Sym1, .., Sym8} offers the best result and reports a significant improvement of 2.21 dB compared to Donoho's standard algorithm involving single wavelet bior 2.6. When using Set4, our approach improves the average SNR_in by 9.02 dB (SNR_out is equal to 19.02 dB).

The mean value of SNR_out using the TI approach is equal to 19.00 dB, compared to 19.02 dB when applying our algorithm using Set4. Both the two methods give practically same results when the taken length is considerably large.

Additionally, it is worthy to note that for shorter lengths of signals for example when using all the 48 records with a length=10s and with SNR_in=10 dB, we remarked that our strategy reached an average SNR_out of 17.57 dB against 16.39 dB related to the TI algorithm.

In terms of complexity, in order to face computational efficiency of our approach against of the TI and RIA algorithms, we can draw up the Table 4. The number of operations for K soft-copies generation needed by our technique is K wavelet denoising operations and an average value calculation operation. Conversely, TI needs $2K$ shift rotations in addition to the operations that our approach requests. For the RIA technique, we need K wavelet denoising operations, $2K$ operations of interpolations and an operation for calculating the mean value. We can state that our approach is the lowest in terms of computational complexity for practically same quality.

Table 4. Number of operations of K soft copies generation.

Operations	**Wavelet denoising**	**Right shift rotation**	**Left shift rotation**	**Interpolation**	**Inverse interpolation**	**Mean value operation**
Proposed method	K	-	-	-	-	1
TI [35]	K	K	K	-	-	1
RIA [29]	K	-	-	K	K	1

We have also assessed and contrasted our method with a recent technique By M. Talbi in 2020 [32] that is regarded as one of the best methods for denoising ECG signals corrupted by white Gaussian noise. The algorithm involves the use of the *stationary bionic wavelet transform* (SBWT) range to apply one-dimensional double-density complex DWT denoising. Simulation results (of seven noised signals 100.dat to 106.dat) in Table 5 are provided from the calculations of SNR_out values and their corresponding MSE for SNR_in range from -5 dB and 15 dB. Each SNR_out represents the average value of seven values of SNR_out related to the cited signals and each MSE is also the mean value of seven MSE computed by using equation (13). The outcomes prove that for all values used of SNR_in, our method performs better than Talbi's Algorithm.

Table 5. Results comparison between our approach and the Talbi's algorithm.

SNR_in (dB)	-5	0	5	10	15
SNR_out (dB) of our approach	**5.65**	**10.15**	**14.72**	**18.77**	**22.24**
SNR_out (dB) of Talbi [32]	5.25	9.71	14.08	18.09	21.66
MSE of our approach	**3.97e-04**	**1.42e-04**	**4.96e-05**	**1.94e-05**	**8.74e-06**
MSE of Talbi [32]	0.0071	0.0026	9.4286e-04	3.7143e-04	1.5714e-04

In Table 6, we also carried out a comparison of our method using Set4 with two techniques based on translation-invariant: Zhang *et al.* [33] and the most recent published paper of Liu *et al.* [34].

Table 6. SNR_out (dB) comparison results of SNR_in=5 dB.

Records	**Zhang *et al.* [33]**	**Liu *et al.* [34]**	**Proposed Method**
100	12.10	-	14.39
103	12.90	-	15.16
105	**14.40**	11.32	14.29
106	-	10.89	**14.61**
107	-	**16.67**	14.29
108	-	9.89	**14.46**
109	-	13.16	**15.03**
111	-	8.67	**13.31**
112	-	**15.01**	13.97
113	12.70	11.70	**15.11**
114	-	8.51	**13.21**
115	13.00	12.90	**15.22**
116	-	**18.65**	14.53
117	13.00	**16.81**	14.90
118	-	**18.16**	13.14
119	12.80	**17.91**	14.60
122	13.50	-	14.44
200	12.70	11.70	**13.80**
201	-	9.02	**14.86**
202	-	8.75	**15.04**
203	-	11.92	**13.96**
205	-	11.08	**14.30**
207	-	10.63	**15.15**
208	-	13.15	**13.86**
209	-	9.76	**13.55**
210	-	9.13	**14.50**
212	-	10.12	**13.51**
215	**13.60**	-	12.99
230	12.00	-	**14.63**
231	-	9.56	**15.06**
232	-	8.65	**14.10**
233	-	14.21	**14.50**
234	-	9.55	**14.28**
Average SNR_out	12.97	12.05	**14.32**

Knowing that the SNR_in is 5 dB, the average SNR_out obtained by Zhang *et al.* is equal to 12.97 dB which is greater than obtained one by Liu *et al.* (12.05 dB). However, our technique offers better results than these two approaches, with SNR_out equal to 14.32 dB.

7. Conclusion

In this paper, a new Translation-invariant-like strategy based on involving a set of wavelets in the denoising process is proposed. The strong point of our work is the generation of noisy soft copies from only one acquired noisy signal, imitating the multisensor acquisition scenario. Additionally, it is less complicated than the TI [35], strategy which uses an additional cycle spinning operation for each soft copy generation and the RIA [29] technique which implies additional interpolation for each soft copy generation. Finally, our method, when compared to the different algorithms reported in [22], [29-35] shows superior performance than the other techniques. Thus, the suggested approach can be considered as an effective, low-cost solution avoiding the use of multisensors to improve noise cancellation. Finally, as a future research direction, powerline interference and baseline wander noises will be reduced by associating to our strategy a technique dedicated to such types of noise.

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