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AN ALTERNATIVE APPROACH FOR DISSEMINATION OF MASS UNIT AFTER THE NEW DEFINITION OF THE KILOGRAM

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Abstract

The paper proposes an alternative approach for the dissemination of the mass unit in the context to the new definition of the kilogram. Considering that redefinition allows mass to be directly realized at any value, the paper presents a model of the dissemination of the mass which can be used for different series in grams, where the measurements are performed in the downward direction, but using 1 g as reference standard (whose mass value is assumed to be determined after the redefinition using the capacitive or electrostatic technique). The subdivision method presented (suitable for E_1 weights) has as starting point the approach used by Mihailov - Romanowsky for the calibration of series in kilograms which uses an orthogonal system of equations. Thus, according to this method, a solution for obtaining the orthogonality of a system can be the use as defining standard of the ratio between the mass having the highest nominal value in the set and the standard (unit). The results obtained for a set of weights from 10 to 1 g using the subdivision method, in accordance with the Mihailov - Romanowsky principle, are compared/validated with those obtained with the multiplication method, where the measurements start from 1 to 10 g, as in the case of the kilograms series. The mass values obtained with these two methods are equal, while the estimated uncertainties are slightly different, but insignificant. The results obtained previously for the same sequence of weights using the traditional dissemination method, where the 1 kg standard is used as reference, are also presented in the paper. The results show that only three weights out of six have an insignificant different mass value by 1×10^{-4} mg compared to those obtained by the methods presented in this article, but, in terms of uncertainty, there are some differences.
The way of disseminating the mass unit presented in this article can be extended to other different sequences of nominal values, such as: (5...1) g, (20...1) g, (50...1) g, or (500…100) g if the reference standard is 100 g.

Keywords: new definition of the kilogram, efficiency of the weighing design, subdivision method, multiplication method, mass dissemination.

1. Introduction

Until November 2018, the kilogram was the single *SI* base unit based on a material artefact. Starting from May 2019, the world said "au revoir" to “*Le Grand K*” that has defined the kilogram for more than one hundred years.

Even if continuing to define the international unit of mass using an object made in the 19th century was considered "scandalous” [1], we cannot fail to appreciate the fact that both the definition and the standard "succeeded together": they were able to withstand the extraordinary evolution that took place in science and technology during the 20th century and the beginning of the 21st century.

The new definition of the kilogram is of particular importance because, the last definition referring to an artefact (the main advantage of the previous definition was its simplicity thus being easily understandable to everyone) was eliminated.

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The new definitions of the four SI base units (the kilogram, the ampere, the kelvin and the mole) came into force as of May 2019 and starting from this date, all SI units have been defined in terms of constants. Thus, the fixed numerical values of the Planck constant (h), the elementary charge (e), the Boltzmann constant (k), and the Avogadro constant (NA) are basis for these new definitions [2].

Currently, there are two main ways which have been taken into consideration to realize the *SI* unit of mass [3]: one of them uses the Kibble balance (the kilogram being defined through the fixed value of Plank constant) and the second one is *X-ray crystal density* (XRCD) method (where the kilogram is defined through the fixed numerical value of Avogadro's number).

Redefinition has allowed mass to be directly realized at any value [4, 5] and at any location using a suitably scaled instrument. This may be an advantage for masses smaller than 1 kg [6]. Thus, with Kibble balances, the mass of weights having different nominal values can be determined (*e.g.*, 50 g to 1 kg) while using other techniques (capacitive, electrostatic) small mass standards (*e.g.*, 10 mg to 1 g) can be realized [7].

The subdivision method presented in this article (suitable especially for E_1 weights) has as starting point the approach used by Mihailov - Romanowsky [8] for the calibration of series in kilograms which uses an orthogonal system of equations. Thus, according to this method, a solution for obtaining the orthogonality of a system can be the use as defining standard of the ratio between the mass having the highest nominal value in the set and the standard (unit).

In this way, the defining standard is attributed to two different masses: (10) which is the highest mass in the set and (1), the lowest one. In the paper, this principle is extrapolated to a group of weights having nominal values from 10 g to 1 g. The steps used for this method are presented in section 2.2. Having the same group of weights and the same equations involved in the calculations, a parallel is made between the results obtained by the two methods: subdivision where measurements start from 10 g to 1 g and multiplication (presented in section 2.3) where measurements start from 1 g to 10 g (as in the case of kilograms series).

One can conclude that the same system of equations prepared for the subdivision method may be as well used for the multiplication method, but the advantage owed to orthogonality is lost as covariances are no longer all equal to zero.

The paper also presents the previous results obtained for the same sequence of weights using the traditional dissemination method (briefly described in section 2.4), where 1 kg standard is used as reference.

This paper may be a good opportunity to choose a new appropriate procedure for the realization of the mass scale, because, even now, the 1 kg mass standard is still used as starting point in the dissemination of the mass unit. The way of disseminating the mass unit presented in this article can be extrapolated to other different sequences of nominal values, such as: (5...1) g, (20...1) g, (50...1) g, *etc.* If the laboratory decides to have as reference mass a weight of 100 g, the method can be used as well, for the sequence of (500…100) g.

2. The mass and the uncertainty determination

2.1. The optimization of the design matrix by evaluating the efficiency of the weighing design

The possible mass comparisons for the interval 10 to 1 are presented in Table 1. Starting from this Table, an efficient design matrix can be chosen, so that the variances of the unknowns are as small as possible.

The efficiency is especially useful when comparing the designs involving the same masses and balances, even if the number of mass comparisons is different. It is desirable that the efficiency of a design be large, as this would indicate that the variances are small [9]. For an efficient scheme, it is advantageous that each mass be used approximately the same number of times and as often as possible. In the scheme, the standard weight is shown in grey whereas the "test" weight(s) is shown in black.

Table 1. Possible mass comparisons between (10…1) g.

Det. No.	**Weights**					
	10	5	2	2*	1	1*
1	–1	1	1	1	1	0
2	–1	1	1	1	0	1
3	0	1	-1	-1	-1	0
4	0	1	-1	-1	0	-1
5	0	0	1	-1	1	-1
6	0	0	1	-1	-1	1
7	0	0	1	-1	0	0
8	0	0	1	0	-1	-1
9	0	0	0	1	–1	-1
10	0	0	0	0	1	-1

To establish the design matrix *X* of the comparisons, several versions were performed and the efficiency of the design for each of them was calculated. As shown in Table 2, using 12 equations (combinations of weights in accordance with Table 1), for the design (1,1,1,1,2,2,0,2,2,0) an efficiency of 1.20 was obtained, while for the design (2,1,0,1,2,0,2,1,2,1) the efficiency obtained was 0.85.

Using 13 equations, for the design (2,1,0,1,0,2,2,2,1,2) an efficiency of about 0.83 was obtained.

In Table 2 the efficiency for different weighing designs is presented and the number of times the weights are used in a design matrix.

Table 2. Efficiency for different weighing design and number of times the weights are used in a design matrix.

Model of matrix design	**Number of times the weights are used**					**Efficiency**
	5	**2**	**2***	**1**	**1***	
12 equations: (1,1,1,1,2,2,0,2,2,0)	4	10	10	10	10	1.20
12 equations: (2,1,0,1,2,0,2,1,2,1)	4	9	10	8	8	0.85
13 equations: (2,1,0,1,0,2,2,2,1,2)	4	10	9	8	8	0.83

Finally, the design (1,1,1,1,2,2,0,2,2,0) was chosen for matrix *X*, having 12 equations of condition. The content of the brackets indicates that the first comparison of Table 1 appears once, the fifth appears twice, *etc.* From the Table 1 one can observe that this model is chosen, since the value for the efficiency is greater, namely 1.20.

Also, it can be seen that for this matrix design, each mass is used the same number of times or as often as possible. In Table 3 the design matrix derived accordingly is presented.

Table 3. The chosen weighing design.

$X =$

Weights				
5	**2**	**2***	**1**	**1***
1	1	1	1	0
1	1	1	0	1
1	-1	-1	-1	0
1	-1	-1	0	-1
0	1	-1	1	-1
0	1	-1	1	-1
0	1	-1	-1	1
0	1	-1	-1	1
0	1	0	-1	-1
0	1	0	-1	-1
0	0	1	–1	-1
0	0	1	–1	-1

The efficiency for this design was determined in the following manner: once all weighings are completed, the first step is to form the design matrix, *X*, which contains the information on the equations used (the weighing design). The vector containing the standard deviation of each comparison (in μg) is represented by *s* and the vector of measured values y_i is represented by *Y*. The elements of β are the values of the unknown departures.

For calculating the efficiency, only standard deviation of each comparison is used.

$$Y = \begin{pmatrix} y_1 + 10r \\ y_2 + 10r \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \end{pmatrix} \quad s = \begin{pmatrix} s_1 = 2.00 \\ s_2 = 0.57 \\ s_3 = 1.50 \\ s_4 = 1.20 \\ s_5 = 0.38 \\ s_6 = 0.38 \\ s_7 = 0.12 \\ s_8 = 0.12 \\ s_9 = 0.25 \\ s_{10} = 0.25 \\ s_{11} = 0.13 \\ s_{12} = 0.13 \end{pmatrix} \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}. \tag{1}$$

Standard deviations of comparisons, represented by *s*, are introduced as diagonal elements in the variance-covariance matrix, denoted by *G*:

$$G = diag(s_1^2, s_2^2 ... s_n^2). \tag{2}$$

Having the design matrix *X*, the matrix *K* can be defined as:

$$K = G^{-1/2} \cdot X \ . \tag{3}$$

Calculating K^T, which is the transpose of *K*, one can determine the inverse $(K^T \cdot K)^{-1}$:

$$(K^{\mathrm{T}}\cdot K)^{-1}=\begin{pmatrix} 0.296 & -0.057 & -0.059 & -0.028 & -0.030 \\ -0.057 & 0.053 & 0.045 & 0.026 & 0.019 \\ -0.059 & 0.045 & 0.049 & 0.022 & 0.025 \\ -0.028 & 0.026 & 0.022 & 0.017 & 0.009 \\ -0.030 & 0.019 & 0.025 & 0.009 & 0.017 \end{pmatrix}. \tag{4}$$

If v_{i} are the diagonal elements of $(K^{\mathrm{T}}\cdot K)^{-1}$ corresponding to the $i^{\text{-th}}$ mass, s_{m} is the largest of the s_{i}, then the efficiency of the design, represented by the matrix X is defined as [9]:

$$E=\Sigma \mathrm{v}_{\mathrm{i}}^{-1}\cdot h_{\mathrm{i}}^{2}\cdot \frac{s_{\mathrm{m}}^{2}}{(\mathrm{n}-1)}, \tag{5}$$

n is the number of comparisons.
h is a k dimensional vector, the i-th element of which is the ratio between the nominal mass of the i-th unknown weight and the reference.

Table 4 presents the calculation of the efficiency for the design (1,1,1,1,2,2,0,2,2,0) containing 12 equations of condition.

Table 4. Calculation (in μg) of the efficiency for the design: (1,1,1,1,2,2,0,2,2,0).

Nominal value	5	2	2*	1	1*	E=1.20
$1/v_{\mathrm{i}}$	3.38	18.81	20.26	60.51	57.80	
h	0.5	0.2	0.2	0.1	0.1	
$h_{\mathrm{i}}^{2}\cdot 1/v_{\mathrm{i}}$	0.85	0.75	0.81	0.60	0.58	
s^{2}_{m}	4.0					
$(h_{\mathrm{i}}^{2}\cdot 1/v_{\mathrm{i}})\cdot s^{2}_{\mathrm{m}}/(\mathrm{n}-1)$	0.28	0.25	0.27	0.20	0.19	
Variance of the weights $V(b)=hh^{t}\,u^{2}_{\mathrm{r}}+v_{\mathrm{i}}$	1.30	0.21	0.21	0.06	0.06	

In the formula of the variance, the first term on the right side represents the contribution to $V(b)$ of each weight due to the variance of the reference and the second term is that resulting from the variances of the mass comparisons, from matrix $(K^{T}\cdot K)^{-1}$.

2.2. The Calibration method for (10…1) g using the Mihailov - Romanowsky principle

A selection rule for a better weighing design (in addition to that presented at section 2.1) should be that the measurements matrix (design matrix) be orthogonal.

Orthogonality is considered to be one of the most important properties for the design of an experiment [10] and, by introducing orthogonality in the mass calibration techniques, the type A uncertainty of the results can be minimized [11].

Even if, in the calibration of weights, the type A uncertainty is a component smaller than others (that are part of the standard uncertainty), it is preferable that its value be as small as possible, to ensure that the design matrix model is the most efficient.

In the mass measurements, when searching for a better design, the objective is to obtain a minimum value of either the variance, if the weights are used independently from each other, or the covariance, if the weights are used in combination. This can be accomplished with an increased number of weighings by addition and repetition of elementary measurements [12].

In Table 3 the design matrix derived for the set 10, 5, 2, 2*, 1, 1* is presented. Some equations from Table 1 are omitted, while others are used more than once, to obtain an orthogonal design. Thus, starting with measurement y_5, all following measurements are repeated two times.

In this calibration, it is assumed that the mass of weight (10) defines a „temporary unit of mass”, (10) = M =10*r* where *r*, equal by definition, to one tenth of the highest mass, (10). Thus, all other masses are now expressed as functions of *r*, using the classical procedure of subdivision.

In a second step, it will be necessary to know *r* in terms of the reference standard, represented by weight (1):

$$10(1) = 10r + N1, \tag{6}$$

*N*1 represents the measurements where the reference standard (1) is used, in accordance to Table 3.

$$N1 = y_1 - y_3 + y_5 + y_6 - y_7 - y_8 - y_9 - y_{10} - y_{11} - y_{12}\ . \tag{7}$$

Having *N*1 calculated, the mass (10) = 10*r* can be easily determined from formula (6), because the value of (1) g reference standard is known from previous calibration.

The value of (10) g (introduced in the vector of *Y* is used to start the calculation of the other weights. Using the least squares method, the unknown values are calculated:

$$\langle \beta \rangle = (X^{\mathrm{T}} \cdot X)^{-1} \cdot X^{\mathrm{T}} \cdot Y\ . \tag{8}$$

The signification of each term from the formula (8) is presented at section 2.1.

$(X^T \cdot X)^{-1}$ is termed the inverse of $(X^{\mathrm{T}} \cdot X)$ and has the next form:

$$(X^{\mathrm{T}} \cdot X)^{-1} = \begin{pmatrix} 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0.10 & 0 & 0 & 0 \\ 0 & 0 & 0.10 & 0 & 0 \\ 0 & 0 & 0 & 0.10 & 0 \\ 0 & 0 & 0 & 0 & 0.10 \end{pmatrix}. \tag{9}$$

Determination of Variances

From formula (6) results:

$$var\ r = var(1) + \left(\frac{1}{10}\right)^2 \cdot var\ N1, \tag{10}$$

where: *var N*1=10·s^2; *s* is the standard deviation of the observations calculated according to formula (14) and *var* (1) is the variance of the reference standard.

Having the variance of *r*, we may calculate the remaining variances as [12, 13]:

$$var(\beta_{\mathrm{j}}) = c_{\mathrm{jj}} \cdot s^2 + h_{\mathrm{j}}^2 \cdot var\ r\ , \tag{11}$$

where: c_{jj} are the diagonal elements of the matrix $(X^{\mathrm{T}} \cdot X)^{-1}$ and the factor $h_j = m_j/m_r$, has the same signification as in formula (5).

Note that formula (11) contains two terms. The first one includes elements of the diagonal matrix, which is the benefit of having chosen an orthogonal design. The second term represents the price we have paid for using the largest weight as a temporary standard.

2.3. The Calibration of (1…10) g using the multiplication method

To ensure the validity of results, according to section 7.7.1, paragraph *f* of [14], a calibration using a different method (by multiplication) was performed.

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Usually, the multiplication method is used for the calibration of kilograms series and consists in comparing the weights starting from the lowest nominal value to the highest nominal value.

In this paper, the method used for kilograms series is extrapolated for (1...10) g using 1 g as reference standard. In addition, this method is used to validate the results obtained in section 2.2, by comparing the mass values and uncertainties resulted for both methods.

The system of equations contains the same measurements combinations of weights as shown in section 2.2. Table 5 contains the weighing design for the multiplication method.

Table 5. The weighing design for (1…10) g using multiplication method.

X=

Weights					
1	**1***	**2***	**2**	**5**	**10**
-1	-1	1	0	0	0
-1	-1	1	0	0	0
-1	-1	0	1	0	0
-1	-1	0	1	0	0
1	-1	-1	1	0	0
1	-1	-1	1	0	0
-1	1	-1	1	0	0
-1	1	-1	1	0	0
-1	0	-1	-1	1	0
0	-1	-1	-1	1	0
1	0	1	1	1	-1
0	1	1	1	1	-1

As shown in section 2.1, the vector of measured values y_i is represented by Y and the elements of β are the values of the unknown departures resulted from the least squares calculation.

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \end{pmatrix} \quad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}. \tag{12}$$

The least squares solutions are calculated as shown in formula (8). The inverse of $(X^{\mathrm{T}} \cdot X)$ has the next form:

$$(X^T \cdot X)^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.2 & 0.2 & 0.2 & 0.5 & 1.0 & 1 \\ 0 & 0.2 & 0.5 & 0.4 & 1.0 & 2.0 & 2 \\ 0 & 0.2 & 0.4 & 0.5 & 1.0 & 2.0 & 2 \\ 0 & 0.5 & 1.0 & 1.0 & 2.7 & 5.0 & 5 \\ 0 & 10.0 & 2.0 & 2.0 & 5.0 & 10.0 & 10 \\ 1 & 1 & 2 & 2 & 5 & 10 & 0 \end{pmatrix}. \tag{13}$$

Determination of Variances

The standard deviation s of the observations is calculated by:

$$s = \sqrt{\frac{1}{\nu}\sum_{i=1}^{n} res_i^2}\ , \tag{14}$$

where:

- the residual res are the elements of the vector $\langle e \rangle$:

$$\langle e \rangle = res_i = Y - \langle Y \rangle\ , \tag{15}$$

$\langle Y \rangle$ is the adjusted mass difference of the weighing equations with:

$$\langle Y \rangle = X \cdot \langle \beta \rangle \quad \text{and} \tag{16}$$

- $\nu = n\text{-}k$ represents the degrees of freedom (with n number of performed observations and k, the number of unknown).

From matrix (13) it can be seen that, for this method, the advantage due to the orthogonality is lost, because the covariances are no longer all equal to zero.

The matrix representing variance – covariance for $\langle \beta \rangle$ is given by:

$$V_\beta = s^2 (X^{\mathrm{T}} \cdot X)^{-1}. \tag{17}$$

For a particular unknown weight, the variance complete is given by:

$$var(\beta_{\mathrm{j}}) = c_{\mathrm{jj}} \cdot s^2 + h_{\mathrm{j}}^2 \cdot var_{\mathrm{ref}}\ , \tag{18}$$

where: c_{ij} are the the diagonal elements of V_β , var_{ref} is the variance associated to reference standard and $h_{\mathrm{j}} = m_{\mathrm{j}}/m_{\mathrm{r}}$, has the same signification as in formula (11). The off-diagonal elements of the matrix V_β give the covariance between the weights.

2.4. The calibration of weights using the traditional dissemination of mass, starting from 1 kg

Even now, the 1 kg mass standard is still used as starting point in the dissemination of mass unit. Thus, in designing the scheme, all the masses from 1 kg to 1 g are broken down into decades. The first decade, containing 12 equations, includes the 1 kg standard. For subsequent decades, the role of the standard is taken by the 1 from the previous decade; thus, the weights having nominal values of 100 g and 10 g masses become intermediate standards, whose uncertainty is propagated directly to masses in the decade they head and hence to those in subsequent decades (each of them containing 12 equations) [15].

Therefore, starting from 1 kg, to calibrate the sequence of (10…1) g, the whole set of weights from (500…1) g should be calibrated. The calibration method as well the calculation of mass and uncertainty of the weights are detailed in many documents, such as [16, 17].

A comparison between the results obtained using this traditional dissemination of mass and those obtained according to sections 2.2 and 2.3 are presented in chapter 3.

3. Numerical example: results

The observed mass differences Y in *mg* (including buoyancy corrections) and the vectors $\langle \beta \rangle$ with the unknown masses, in *mg*, calculated according to formula (8), for both methods (presented at section 2.2 and 2.3) are given below.

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Subdivision-Orthogonal method

$$Y = \begin{pmatrix} y_1 = 0.0214 \\ y_2 = 0.0214 \\ y_3 = -0.0152 \\ y_4 = -0.0135 \\ y_5 = -0.0006 \\ y_6 = -0.0006 \\ y_7 = -0.0052 \\ y_8 = -0.0052 \\ y_9 = -0.0159 \\ y_{10} = -0.0159 \\ y_{11} = -0.0132 \\ y_{12} = -0.0132 \end{pmatrix}. \langle \beta \rangle = \begin{pmatrix} 5\text{g} = 0.0035 \\ 2\text{g} = 0.0028 \\ 2\text{g}^* = 0.0057 \\ 1\text{g}_{\text{ref}} = 0.0104 \\ 1\text{g} = 0.0084 \end{pmatrix}$$

Multiplication method

$$Y = \begin{pmatrix} y_1 = -0.0159 \\ y_2 = -0.0159 \\ y_3 = -0.0132 \\ y_4 = -0.0132 \\ y_5 = -0.0006 \\ y_6 = -0.0006 \\ y_7 = -0.0052 \\ y_8 = -0.0052 \\ y_9 = -0.0152 \\ y_{10} = -0.0135 \\ y_{11} = 0.0044 \\ y_{12} = 0.0044 \end{pmatrix} \langle \beta \rangle = \begin{pmatrix} 10\text{g} = 0.0170 \\ 5\text{g} = 0.0035 \\ 2\text{g} = 0.0028 \\ 2\text{g}^* = 0.0057 \\ 1\text{g}_{\text{ref}} = 0.0104 \\ 1\text{g} = 0.0084 \end{pmatrix}.$$

For the subdivision-orthogonal method, the mass of 10 g calculated according to formula (6), has a value of 0.0170 mg.

A centralization of the results containing mass values and standard uncertainty obtained for the methods described in sections 2.2 and 2.3 are presented in Table 6. The results obtained using traditional method of dissemination, according to section 2.4, are also included in this table.

Table 6. Mass and standard uncertainty obtained for each method.

Weight	**Subdivision-Orthogonal method**		**Multiplication method**		**Traditional method starting from 1 kg**	
Nominal value	β (mg)	u (k=1) (mg)	β (mg)	u (k=1) (mg)	β (mg)	u (k=1) (mg)
10 g	0.0170	0.0027	0.0170	0.0030	0.0170	0.0010
5 g	0.0035	0.0014	0.0035	0.0015	0.0034	0.0005
2 g	0.0028	0.0006	0.0028	0.0006	0.0028	0.0003
2* g	0.0057	0.0006	0.0057	0.0006	0.0056	0.0003
1g $_{\text{ref}}$	**0.0104**	**0.0002**	**0.0104**	**0.0002**	0.0104	0.0002
1* g	0.0084	0.0003	0.0084	0.0004	0.0083	0.0002

In order to compare the performance of the different methods, the uncertainty reported in Table 6 was evaluated with the next contributions: reference weight, weighing process and resolution of the mass comparator (this component was added to formula 11 and 18). For the determinations containing the weights of 10 g and 5 g, a mass comparator with a resolution of 1 µg was used, while for the other measurements, a mass comparator with a resolution of 0.1 µg was used. Uncertainty contribution associated to buoyancy correction was negligible.

Analysing the results from the Table 6, one can see that the mass values obtained using subdivision-orthogonal method, are equal to those values obtained from the multiplication method, having only some insignificant differences in uncertainty estimation. Also, the results obtained by the traditional method show that only three weights out of six have an insignificant different mass value by $1\text{x}10^{-4}$ mg compared to those obtained by the methods described at sections 2.2 and 2.3, but, in terms of uncertainty, there are some differences, especially at 10 g and 5 g: 0.0017 mg, respectively 0.0009 mg.

Figure 1 shows the results obtained for each calibration weight (mass and standard uncertainty).

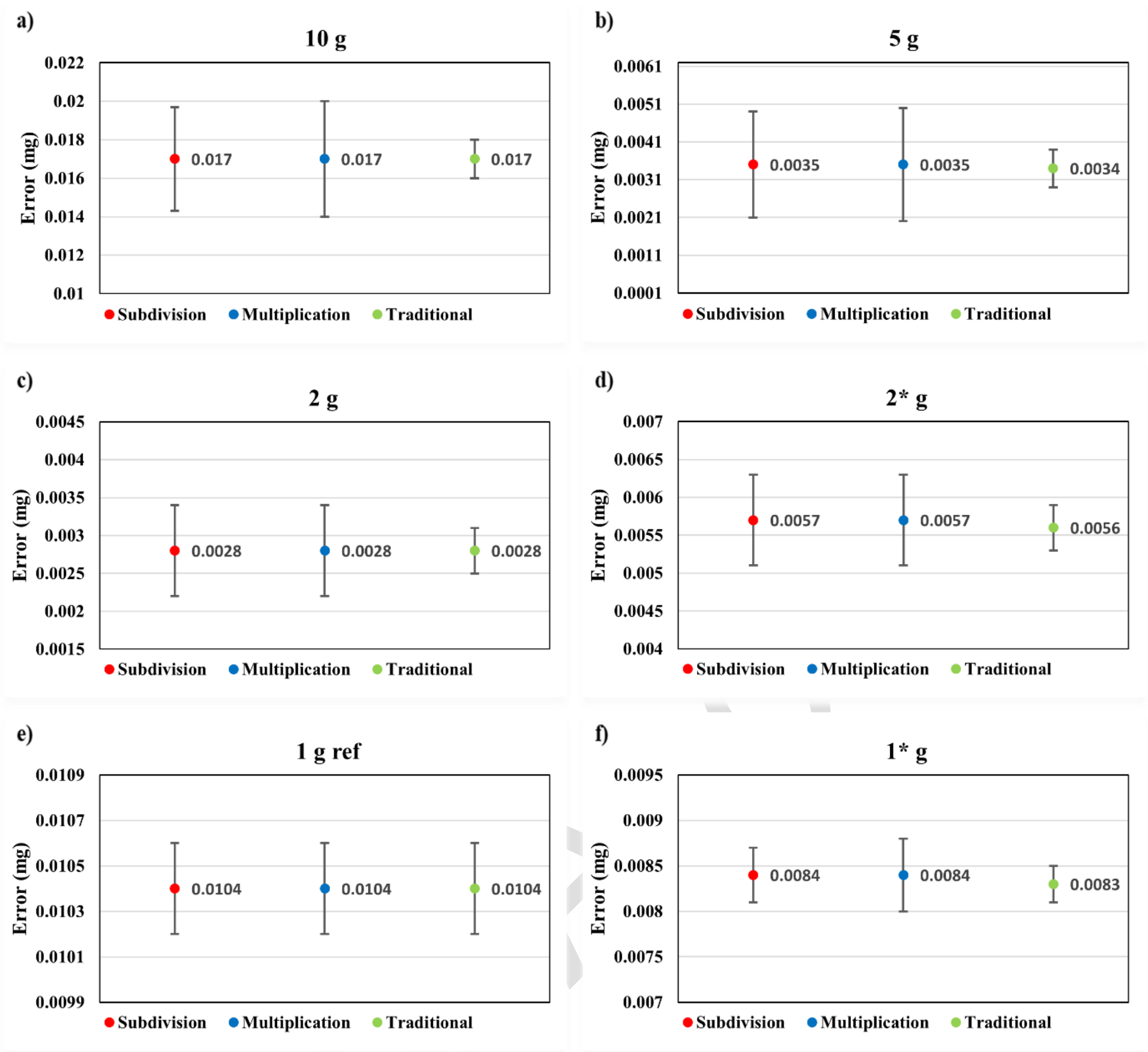

Fig. 1 Mass and standard uncertainty for: a) 10 g, b) 5 g, c) 2 g, d) 2* g, e) 1g ref and f) 1* g.

In Figs. 2 and 3, the correlation graph between the results (mass and uncertainty) from the Subdivision, Multiplication and Traditional method is presented. Both, the graphs and the value of the correlation coefficient, *r* (with *r*=0.995...1), indicate that there is a perfect (or strong) positive linear relationship between the results.

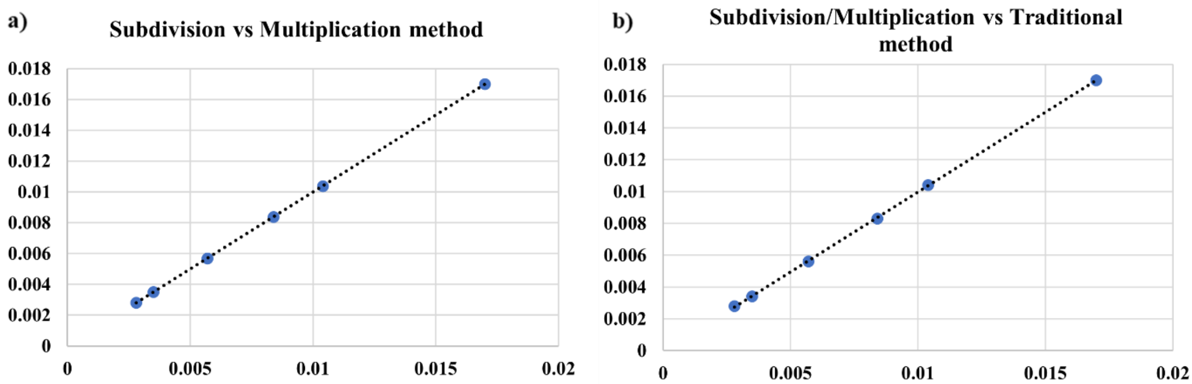

Fig. 2. The correlation graph between the mass results from: (a) Subdivision vs Multiplication and (b) Subdivision/Multiplication vs Traditional method, (*r*=1).

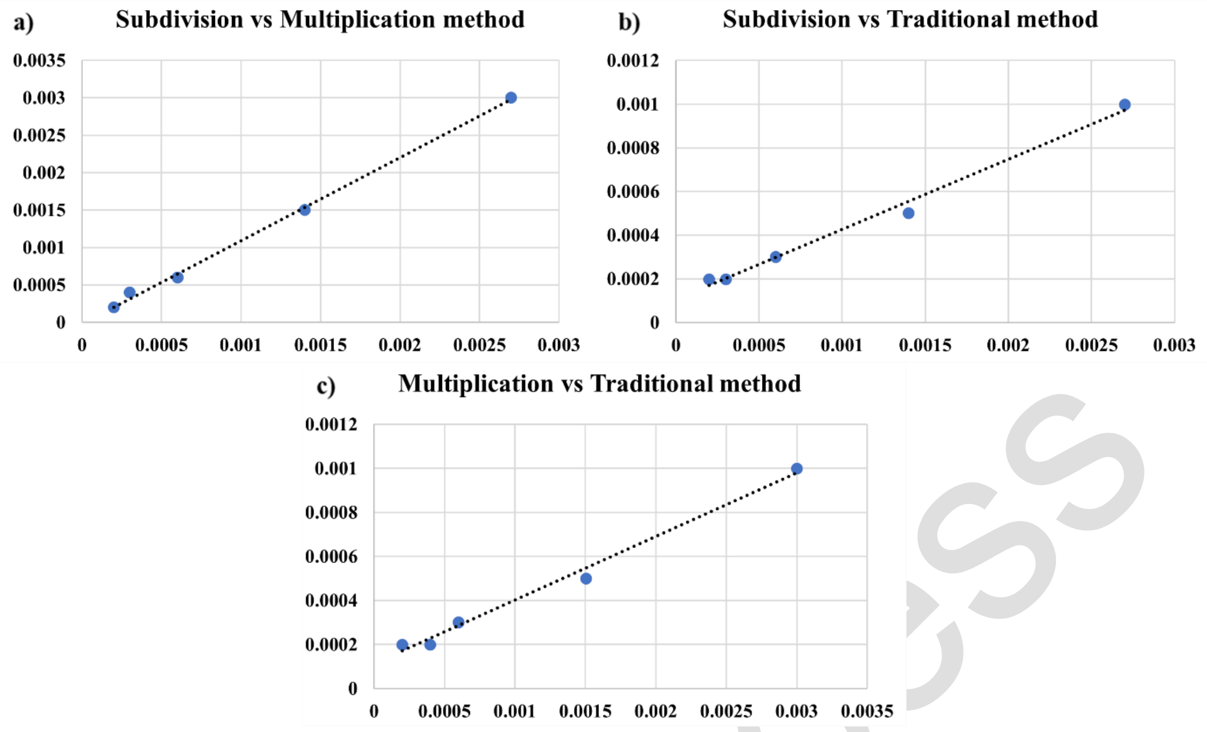

Fig. 3. The correlation graphs between the uncertainty results from: (a) Subdivision vs Multiplication, (b) Subdivision vs Traditional and (c) Multiplication vs Traditional, (*r*=0.995…0,999).

4. Conclusions

The paper proposes an alternative approach for the dissemination of the mass unit in the context to the new definition of the kilogram.

Considering that the redefinition allows the mass to be directly realized at any value and at any location using an appropriately scaled instrument, a model of the mass dissemination for the series in grams (especially suitable for class E_1 weights) is presented in the paper using an adaptive subdivision method, having 1 g as reference standard (whose mass value is assumed to be determined after the redefinition). To ensure the validity of results, according to [14], a calibration using a different method (by multiplication) was performed.

A comparison between the results obtained for the three methods: subdivision, multiplication and traditional dissemination of mass are presented in Table 6.

By analysing the results, one can see that the mass values obtained using subdivision-orthogonal method, are equal to those values obtained from the multiplication method, having only some insignificant differences in the estimation of the uncertainty.

Comparing these mass values and uncertainties with those achieved in traditional dissemination from the „old kilogram", only three weights out of six have an insignificant different mass value by 1×10^{-4} mg, but, in terms of uncertainty, there are some differences, especially at 10 g and 5 g: 1.7×10^{-3} mg, respectively 9×10^{-4} mg.

The results are also represented in the correlation graphs. Both, the graphs and the value of the correlation coefficient, *r* (with *r*=0.995...1), indicate that there is a perfect or strong positive linear relationship between the results.

From formulas 11 and 18 one can see that an improvement in the standard deviation of the weighing process could lead to a decrease in the value of the uncertainty for the weights. This could be achieved by increasing the number of weighings, especially for measurements where the weights of 10 g and 5 g are involved. If possible, another way to reduce the standard uncertainty of the weights could be performing at least the sequence of measurements from 5 g to 2 g on the same mass comparator, having the resolution of 0.1 μg.

Even if some differences were obtained in terms of uncertainty, in the calibration certificate, the results should be reported according to chapter 5.2. of [18], according to which, the expanded uncertainty, *U*, for k = 2, of the conventional mass, shall be less than or equal to one-third of the maximum permissible error. Thus, from this point of view, one can consider that this condition was met for all methods.

The main advantage of this adaptive subdivision method is the fact that the number of measurements is substantially reduced (compared to the traditional method) because only 12 equations corresponding to the sequence of (10…1) g are used, whereas the previous 24 equations are no longer performed.

The way of disseminating the mass unit presented in this article can be extended to other different sequences of nominal values, such as: (5...1) g, (20...1) g, (50...1) g, or (500…100) g if the reference standard is 100 g.

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5. References

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